MATT BECK, San Francisco State University
Symmetrically constrained compositions

The study of partitions and compositions (i.e., ordered partitions) of integers goes back centuries and has applications in various areas within and outside of mathematics. Partition analysis is full of beautiful—and sometimes surprising—identities. As an example (and the first motivation for our study), we mention compositions $(\lambda_1, \lambda_2, \lambda_3)$ of an integer m (i.e., $m = \lambda_1 + \lambda_2 + \lambda_3$ and all $\lambda_j \in \mathbf{Z}_{\geq 0}$) that satisfy the six "triangle conditions"

$$\lambda_{\pi(1)} + \lambda_{\pi(2)} \geq \lambda_{\pi(3)} \qquad \text{for every permutation } \pi \in S_3 \, .$$

George Andrews proved in the 1970's that the number $\Delta(m)$ of such compositions of m is encoded by the generating function

$$\sum_{m \geq 0} \Delta(m) \, q^m = \frac{1}{(1-q^2)^2(1-q)} \, .$$

More generally, for fixed given integers $a_1, a_2, \ldots, a_n$, we call a composition $\lambda_1 + \lambda_2 + \cdots + \lambda_n$ *symmetrically constrained* if it satisfies each of the the $n!$ constraints

$$\sum_{j=1}^{n} a_j \lambda_{\pi(j)} \geq 0 \qquad \text{for every permutation } \pi \in S_n \, .$$

We show how to compute the generating functions of these compositions, combining methods from partition theory, permutation statistics, and polyhedral geometry. This is joint work with Ira Gessel, Sunyoung Lee, and Carla Savage.