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A refinement of the Kolmogorov–Marcinkiewicz–Zygmund strong law of large numbers

Let $\{X_n\,; n \geq 1\}$ be a sequence of independent copies of a real-valued random variable X and set $S_n = X_1 + \cdots + X_n$, $n \geq 1$. This paper is devoted to a refinement of the classical Kolmogorov–Marcinkiewicz–Zygmund strong law of large numbers. We show that for $0 < p < 2$,

$$\sum_{n=1}^{\infty} \frac{1}{n}\Big(\frac{|S_n|}{n^{1/p}}\Big) < \infty \quad \text{almost surely}$$

if and only if

$$\begin{cases} \mathbb{E}|X|^p < \infty & \text{if } 0 < p < 1, \\ \mathbb{E}X = 0, \sum_{n=1}^{\infty} \frac{\left|\mathbb{E}XI\{|X| \leq n\}\right|}{n} < \infty, & \text{and} \\ \sum_{n=1}^{\infty} \frac{\int_{\min\{u_n, n\}}^{n} \mathbb{P}(|X| > t)\, dt}{n} < \infty & \text{if } p = 1, \\ \mathbb{E}X = 0 \text{ and } \int_0^{\infty} \mathbb{P}^{1/p}(|X| > t)\, dt < \infty & \text{if } 1 < p < 2, \end{cases}$$

where $u_n = \inf\{t : \mathbb{P}(|X| > t) < \frac{1}{n}\}$, $n \geq 1$. Versions of above results in a Banach space setting are also presented. To establish these results, we invoke the remarkable Hoffmann–Jørgensen inequality to obtain new maximal inequality which may be of independent interest but which we apply to $\sum_{n=1}^{\infty} \frac{1}{n}\big(\frac{|S_n|}{n^{1/p}}\big)$.

This talk is based on a recent paper by me and Professors Yongcheng Qi and Andrew Rosalsky.