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Amenability of $\mathcal{B}(\ell^p)$ for $p \neq 2$

In 1972, B. E. Johnson asked if $\mathcal{B}(E)$, the Banach algebra of all bounded linear operators on a Banach space E could be amenable unless $\dim E < \infty$. It follows from work of A. Connes and S. Wassermann that $\mathcal{B}(\ell^2)$ cannot be amenable. Only recently, efforts by C. J. Read, G. Pisier, and N. Ozwa showed that $\mathcal{B}(\ell^p)$ is also not amenable if $p = 1, \infty$. In this talk, which is based on joint work with M. Daws, we explore the consequences of the hypothetical amenability of $\mathcal{B}(\ell^p)$ for $p \in (1,\infty) \setminus \{2\}$. In particular, we show that the amenability of $\mathcal{B}(\ell^p)$ implies the *ultra-amenability* of $\mathcal{K}(E)$ for every $\mathcal{L}^p$-space E, i.e., $\big(\mathcal{K}(E)\big)_{\mathcal{U}}$ is amenable for every ultrafilter $\mathcal{U}$. For instance, the amenability of $\mathcal{B}(\ell^p)$ implies the ultra-amenability of $\mathcal{K}(\ell^p \oplus \ell^2)$; this is remarkable because $\mathcal{B}(\ell^p \oplus \ell^2)$ is known to be non-amenable.