Algebraic Combinatorics, Representations and Geometry
Combinatoire algébrique, représentations et géométrie
(Org: **Lex Renner** (Western) and/et **Benjamin Steinberg** (Carleton))

MARCELO AGUIAR, Texas A&M University, College Station, TX
Monoidal categories, Joyal's species, and combinatorial Hopf algebras

We provide a categorical framework for the construction and study of combinatorial Hopf algebras. Two central notions are Joyal's species and bilax monoidal functors. A species is a combinatorial version of a graded vector space. A good example of a bilax monoidal functor arises in topology, in the context of simplicial sets and chain complexes: the bilax structure is afforded by the Alexander–Whitney and Eilenberg–Zilber maps. We discuss analogous functors in the context of species and graded vector spaces that form the basis for our applications to combinatorial Hopf algebras. The categorical approach yields uniform deformations and higher dimensional generalizations of these objects. We encounter at this point a remarkable connection between species and combinatorial Hopf algebras on the one hand, and quantum groups and the classification theory of abstract Hopf algebras on the other.

This is part of joint work with Swapneel Mahajan.

MAHIR CAN, University of Western Ontario
Bruhat Orders and Combinatorics on Reductive Monoids

Originating from the early '80s, the theory of reductive monoids is a combinatorial, fledgling branch of algebraic geometry within the theory of spherical embeddings. It brings together algebraic groups, the torus embeddings and semigroups. One can (naïvely) describe a reductive monoid M as the Zariski closure of the image of a representation $\sigma\colon G \to \mathrm{End}(V)$ of a reductive group in the $\mathrm{End}(V)$.

With this description, many pleasant features of the group G lift up to M, however the catch is the set of idempotents which, of course, lacks from the group structure. In this talk we shall concentrate on the generalized Bruhat ordering on M. In the case of $n \times n$ matrices, we shall give a purely combinatorial characterization of the Bruhat ordering (in the group case, it is originally due to V. Deodhar). We shall also give combinatorial formulas for the dimensions of the $B \times B$ orbits in M. If time permits, we shall describe an analogue of the Hasse–Weil zeta function for M and give a recipe to compute it.

This is a joint work with Prof. Renner.

JOHN FOUNTAIN, University of York, York YO10 5DD, UK
Reflection Monoids

The inverse monoid of all partial linear isomorphisms a vector space V is denoted by $ML(V)$. A *partial reflection* is defined to be the restriction of a reflection of V to a subspace of V, and a *reflection monoid* is a factorisable inverse submonoid of $ML(V)$ generated by partial reflections. A reflection monoid can be characterised by two pieces of data: a reflection group W on V and a collection of subspaces of V that forms a W-invariant semilattice and contains V itself. In the talk we will outline the basic properties of reflection monoids and give some examples. We also mention connections with Renner monoids, and give the orders of, and presentations for, some of the monoids.

EDDY GODELLE, Université de Caen, LMNO 14032 Caen Cedex, France
The braid rook monoid

Let W be a finite monoid and $\ell\colon W \to \mathbb{N}$ be a map. One can consider the monoid B defined by the presentation

$$\langle \underline{w} \in \underline{W} \mid \underline{ww'} = \underline{ww'} \text{ when } \ell(ww') = \ell(w) + \ell(w')\}$$

where $\underline{W}$ is a formal copy of W. If W is the permutation group on n elements and ℓ is its standard length function, one obtains the monoid of positive braids on n strands. Here I consider the case where W is the rook monoid, which a natural generalization of the permutation group.

CHRISTOPHE HOHLWEG, Université du Québec à Montréal, Département de Mathématiques, CP 8888, Succ. Centre-Ville, Montréal, Québec H3C 3P8
Permutahedra and Generalized Associahedra

Given a finite Coxeter system (W, S) and a Coxeter element c, we construct a simple polytope whose outer normal fan is N. Reading's Cambrian fan $\mathcal{F}_c$, settling a conjecture of Reading that this is possible. We call this polytope the c-generalized associahedron. These polytopes are combinatorially isomorph to the generalized associahedra related to cluster algebras and introduced by S. Fomin and A. Zelevinsky.

Our approach generalizes Loday's realization of the associahedron (a type A c-generalized associahedron whose outer normal fan is not the cluster fan but a coarsening of the Coxeter fan arising from the Tamari lattice) to any finite Coxeter group. A crucial role in the construction is played by the c-singleton cones, the cones in the c-Cambrian fan which consist of a single maximal cone from the Coxeter fan.

ZHENHENG LI, University of South Carolina Aiken, 471 University Parkway, Aiken, SC 29801
Representations of the Symplectic Renner Monoid

In this talk we concern representations of the symplectic Renner monoid R. We determine irreducible representations of R in terms of the irreducible representations of certain symmetric groups and those of the symplectic Weyl group W. We then give the character formula of R using the character of W and that of the symmetric groups.

ZHUO LI, Xiangtan University, Xiangtan, Hunan, China 411105
Poincaré Polynomials of Combinatorially Smooth Toric Varieties

Let M be a $\mathcal{J}$-irreducible monoid associated with a integral polytope P over an algebraically closed field K. Let T be its maximal torus. Then $\overline{T} \setminus \{0\}/K^*$ can be identified as the toric variety X_P constructed from the polytope P. The toric variety X_P is called combinatorially smooth if the polytope P is simple. In this talk, we describe a very powerful theorem by Renner to determine when the toric variety X_P is combinatorially smooth. Then we try to find the Poincaré polynomials of such toric varieties (or equivalently f-vectors or h-vectors of the polytopes, or Hilbert series of the face rings of the polytopes). It turns out that the Renner monoids hold all the information to calculate the Poincaré polynomials.

MARTIN MALANDRO, Dartmouth College, Hanover, NH, USA
A Fast Fourier Transform for the Rook Monoid

We define the notion of the Fourier transform for a finite inverse semigroup S and we address the problem of computing it in a time-efficient manner for $S = R_n$, the rook monoid (also known as the symmetric inverse semigroup) on n elements. We do so by exploiting recently developed tools in semigroup theory, and we give an indication as to how these tools generalize to create fast Fourier transforms for arbitrary finite inverse semigroups.

CLAUS MOKLER, University of Wuppertal, Gaußstrasse 20, 42097 Wuppertal, Germany
The face monoid associated to a Kac–Moody group and its infinite Renner monoid

The face monoid and its coordinate ring are obtained from the category of integrable representations of the category O of a symmetrizable Kac–Moody algebra by a Tannaka reconstruction. The face monoid contains the Kac–Moody group as open dense unit group. Its idempotents are related to the faces of the Tits cone. It has similar structural properties as a reductive algebraic monoid, but its Renner monoid is infinite. In my talk I give some results on:

(a) Actions of the Renner monoid on the Coxeter complex of the Weyl group and induced actions of the face monoid on the building of the Kac–Moody group.

(b) The parabolic partition of the Renner monoid, the parabolic partition and the adjoint quotient map of the face monoid.

(c) The combinatorics of the order relations on the Renner monoid, which are obtained by the closure relations of the Bruhat and Birkhoff cells.

LEX RENNER, University of Western Ontario, London, Ontario
Betti Numbers and H-polynomials

The Poincaré polynomial of a Weyl group calculates the Betti numbers of G/B. The h-vector of a rational, simplicial polytope calculates the Betti numbers of a corresponding toric variety. There is a common generalization of these two extremes called the *H-polynomial*. It applies to projective, homogeneous spaces, toric varieties and, much more generally, any algebraic variety X where there is a connected, solvable, algebraic group acting with a finite number of orbits. We illustrate the situation by calculating some H-polynomials related to generalized "rook" monoids.

FRANCO SALIOLA, Université du Québec à Montréal
Left regular bands and Solomon's descent algebras

Inside the group algebra of a *finite Coxeter group* lives a highly exceptional subalgebra called the *descent algebra*, which has connections with combinatorics, representation theory, Lie algebras and random walks, to name a few. It has recently been described in geometric and semigroup theoretic terms as a subalgebra of the semigroup algebra of a *left regular band*. (The left regular band is the Coxeter complex with a geometrically defined product, and the subalgebra is the subalgebra that is invariant under the action of the Coxeter group.) This talk will describe how this semigroup can be used to understand the structure of the descent algebra, and how this leads to a description of the *quiver* of the descent algebra in type A and type B.

BENJAMIN STEINBERG, Carleton University, 1125 Colonel By Drive, Ottawa, ON
On semigroups with basic complex algebra

Semigroups with a basic complex algebra can be characterized algebraically in several different ways. One such is that they have a homomorphism to a commutative inverse semigroup inducing the semisimple quotient. Consequently computing multiplicities of irreducible constituents reduces to the case of a commutative inverse semigroup. This can be handled combinatorially using Mobius inversion on the semilattice of idempotents and inner product formulas for abelian groups. Consequently the spectrum of a Random Walk on any such semigroup can be explicitly computed. This generalizes results of Ken Brown and others.

HUGH THOMAS, University of New Brunswick, Fredericton, NB E3B 5A3
A new partial order on a finite reflection group

I will discuss a new partial order on a finite reflection group W which has recently been studied by Drew Armstrong, based on a construction due to Nathan Reading which initially appeared in connection with the combinatorics of cluster algebras. This order (which depends on a choice of Coxeter element for W) is stronger than weak order, but weaker than Bruhat order. Like weak order, this order can be defined by associating to any $w \in W$ a set of positive roots and then considering the inclusion order on those sets, but the sets of positive roots that appear are not generally inversion sets and do not seem to have been studied before. I will present some results describing these sets of positive roots, using the theory of quiver representations.

This is joint work with Drew Armstrong (University of Minnesota).

HAMID USEFI, University of British Columbia, 1984 Mathematics Road, Vancouver, BC V6T 1Z2, Canada
Fox subgroups in modular group algebras

Let G be a finite p-group, S a subgroup of G, and F the prime field of characteristic p. We denote the augmentation ideal of the group algebra FG by $\omega(G)$. The Zassenhaus–Jennings–Lazard series of G is defined by $D_n(G) = G \cap \big(1 + \omega^n(G)\big)$. We first recall a theorem of Quillen stating that the graded algebra associated to FG is isomorphic as an algebra to the enveloping algebra of the restricted Lie algebra associated to the $D_n(G)$. We then extend a theorem of Jennings that provides a basis for the quotient $\omega^n(G)/\omega^{n+1}(G)$ in terms of a basis of the restricted Lie algebra associated to the $D_n(G)$. We shall characterize the subgroups $G \cap \big(1 + \omega(G)\omega^n(S)\big)$ and $G \cap \big(1 + \omega^2(G)\omega^n(S)\big)$, for every positive integer n. These are the modular analogues of Fox subgroups in integral group rings of free groups.

STEVEN WANG, Carleton University
Schur rings and planar functions

Earlier, Schur rings have been found useful in the construction of association schemes which are important combinatorial objects with applications in other areas of research as well. Recently, Schur rings are a powerful tool for solving Cayley graph isomorphism problem. For Cayley graphs over p-groups a great role is played by p-Schur rings. In general, a Schur ring is not necessary Schurian. We show that a p-Schur ring over an elementary abelian group of rank 3 is always Schurian by using a well known result of planar functions over prime fields. This answers a question of Hirasaka and Muzychuk.

This paper is a joint work with Pablo Spiga.