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*Indefinite Contractions*

Given a matrix $A$, define the triple $(\pi_-(A), \pi_0(A), \pi_+(A))$ as its inertia with respect to the unit circle where $\pi_-(A)$ (resp. $\pi_+(A)$) is the number of eigenvalues of $A$ inside (resp. outside) the unit disc while $\pi_0(A)$ is the number of eigenvalues on the unit circle.

An invertible Hermitian matrix $H$ gives rise to an (indefinite) inner product. A matrix $A$ is called an $H$-(strict) contraction (or $H$-contractive) if $H > A^*HA$. The most interesing case is that $H$ is an involution, $H^2 = I$. We use $J$ for $H$ in such a case.

It is well known that a matrix $A$ is $H$-contractive for suitable $H$ if and only if $\pi_0(A) = 0$. Though $H$ is not determined uniquely by $A$ (even up to positive scalar multiple), $\pi_-(A)$ must coincide with the number of positive eigenvalues of $H$.

If $A$ is $J$-contractive, so is $A^*$ and hence $A^*A$. Therefore $A$ and its modulus $|A| \equiv (A^*A)^{1/2}$ have the same inertia. But this property does not seem sufficient to guarantee $J$-contractivity of $A$ for a suitable involution $J$. Other necessary conditions are presented.

If $H$-contractivity of a matrix $A$ always guarantees that of its adjoint $A^*$ then $H$ is necessarily a scalar multiple of an involution.

A characterization is given for a set of matrices coincides with the set of $H$-contractions for (unknown) $H$.