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Fast iterative Gaussian quadrature

We construct a family $\mathcal{F}$ of probability distributions on the real line for which iterated Gaussian quadrature, where the number of nodes is approximately doubled at each iteration, is computationally more efficient than usual. For each $d\alpha$ in $\mathcal{F}$, the $2n+1$-point Gauss rule re-uses all the nodes of the n-point Gauss rule, the $2(2n+1)+1$-point rule re-uses the nodes of the $2n+1$-point rule, and so on indefinitely. We show that it is possible to construct a distribution of this type for an essentially arbitrary sequence of nodes. This implies, for example, the existence of a distribution supported on $[-1,1]$ whose n-point Gauss rules have evenly spaced nodes, and (equivalently) whose orthogonal polynomials of degree n have evenly spaced zeros, for an unbounded sequence of indices n.

In addition we give an explicit construction for the subclass $\mathcal{OF}$ of $\mathcal{F}$ for which iteration of Gaussian quadrature re-uses, not only the nodes, but also the weights at each step. The classical distribution $\sqrt{1-x^2}\,dx$ is derived as a particular example.