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When Systems Collide

We present a conjecture which is a common generalization of the Doyen–Wilson Theorem and Lindner and Rosa's intersection theorem for Steiner triple systems. Given $u, v, \equiv 1, 3 \pmod 6$, $u < v < 2u+1$ we ask for the minimum r such that there exists a Steiner triple system $(U, \mathcal{B})$, $|U| = u$ such that some partial system $(U, \mathcal{B} \setminus \mathfrak{d})$ can be completed to a STS(v), $(V, \mathcal{B}')$, where $|\mathfrak{d}| = r$. In other words, in order to "quasi-embed" an STS(u) into an STS(v), we must remove r blocks from the small system, and this r is the least such with this property. One can also view the quantity $u(u-1)/6 - r$ as the maximum intersection of an STS(u) and an STS(v) with $u < v$. We conjecture that the necessary minimum $r = (v-u)(2u+1-v)/6$ can be achieved, except when $u = 6t+1$ and $v = 6t+3$, in which case it is $r = 3t$ for $t \neq 2$, or $r = 7$ when $t = 2$. Using small examples and recursion, we solve the cases $v - u = 2$ and 4, asymptotically solve the cases $v - u = 6, 8, 10$, and further show for given $v - u > 2$ that an asymptotic solution exists if solutions exist for a run of consecutive values of u (whose required length is no more than $v - u$). Some results are obtained for v close to $2u + 1$ as well. The cases where $v \approx 3u/2$ seem to be the hardest. For intersections sizes between 0 and this maximum we generalize Lindner and Rosa's *intersection problem*—"determine the possible numbers of blocks common to two Steiner triple systems STS(u), $(U, \mathcal{B})$, $U = V$" to the cases STS(v), $(V, \mathcal{B}')$, with $U \subseteq V$ and solve it completely for $v - u = 2, 4$ and for $v \geq 2u - 3$.

Joint work with P. Dukes.