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Oriented Matroids as Combinatorial Structures Underlying Loop Quantum Gravity

We analyze combinatorial structures which play a central role in determining spectral properties of the volume operator in loop quantum gravity (LQG). These structures encode geometrical information of the embedding of arbitrary valence vertices of a graph in 3-dimensional Riemannian space, and can be represented by sign strings containing relative orientations of embedded edges. We demonstrate that these signature factors are a special representation of the general mathematical concept of an oriented matroid. Moreover, we show that oriented matroids can also be used to describe the topology (connectedness) of directed graphs. Hence the mathematical methods developed for oriented matroids can be applied to the difficult combinatorics of embedded graphs underlying the construction of LQG. As a first application we revisit our earlier analysis of the volume spectrum, and find that the enumeration of all possible sign configurations used there is equivalent to enumerating all realizable oriented matroids of rank 3, and thus can be greatly simplified. We find that for 7-valent vertices having no coplanar triples of edge tangents, the smallest non-zero eigenvalue of the volume spectrum does not grow as one increases the maximum spin jmax at the vertex, for any orientation of the edge tangents. This indicates that, in contrast to the area operator, considering large jmax does not necessarily imply large volume eigenvalues. In addition we give an outlook to possible starting points for rewriting the combinatorics of LQG in terms of oriented matroids.