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Combinatorial Hopf algebras and Towers of Algebras—dimension, quantization, and functoriality

With N. Bergeron, I have introduced a set of axioms which guarantee that the Grothendieck groups of a tower of algebras $\bigoplus_{n\geq 0} A_n$ can be endowed with the structure of graded dual Hopf algebras. Hivert and Nzeutzhap, and independently Lam and Shimozono, constructed dual graded graphs from primitive elements in Hopf algebras. With N. Bergeron and T. Lam, I apply the composition of these constructions to towers of algebras. We show that if a tower $\bigoplus_{n\geq 0} A_n$ gives rise to graded dual Hopf algebras then we must have $\dim A_n = r^n n!$ where $r = \dim A_1$. This shows that combinatorial Hopf algebras obtained by this procedure fall into a very rigid framework and can potentially be classified. In the case $r = 1$ we give a conjectural classification. We then investigate a quantum version of the main theorem. We conclude with some open problems and a categorification of the construction.