
MEGUMI HARADA, McMaster University

Hyperkähler Kirwan surjectivity for quiver varieties: Morse theory and examples

Let G be a compact Lie group. The well-known Kirwan surjectivity theorem in equivariant symplectic geometry states that the G -equivariant rational cohomology of a Hamiltonian G -space (M, ω) surjects onto the ordinary rational cohomology of the symplectic quotient of M by G . This surjective ring homomorphism (“the Kirwan map”) has been a key tool in computations of the topology of symplectic quotients.

I will discuss our recent progress on the analogous hyperkähler question, namely: if $(M, \omega_1, \omega_2, \omega_3)$ is a hyperkähler hyperhamiltonian G -space, then does the G -equivariant cohomology of M surject onto the ordinary rational cohomology of the hyperkähler quotient of M by G ? We restrict to the case of Nakajima quiver varieties and develop a Morse theory for the hyperkähler moment map analogous to the case of the moduli space of Higgs bundles. In particular, we show that the Harder–Narasimhan stratification of spaces of representations of quivers coincide with the Morse-theoretic stratification associated to the norm-square of the real moment map.

Our approach also provides insight into the topology of specific examples of small-rank quiver varieties, including hyperpolygon spaces and some ADHM quivers.

This is a preliminary report on work in progress with Graeme Wilkin.