GERMAN DZYUBENKO, International Mathematical Center of NAS of Ukraine, 01601, Tereschenkivska Str., 3, Kyiv-4, Ukraine

Shape preserving approximation of periodic functions

Let $2s$, $s \in \mathbb{N}$, fixed points y_i $-\pi \le y_{2s} < y_{2s-1} < \cdots < y_1 < \pi$ are given and for the other indexes $i \in \mathbb{Z}$, the points y_i are defined periodically, i.e., by the equality $y_i = y_{i+2s} + 2\pi$, $Y := \{y_i\}_{i \in \mathbb{Z}}$. ¿From the space C of continuous 2π-periodic functions $f\colon \mathbb{R} \to \mathbb{R}$ with the norm $\|f\| := \max_{x\in\mathbb{R}} |f(x)|$, we extract three sets $\Delta^{(q)}(Y)$, $q = 0, 1, 2$, of all functions f which are, respectively, nonnegative/nondecreasing/convex on $[y_1, y_0]$, nonpositive/nonincreasing/concave on $[y_2, y_1]$ and so on. Let

$$E_n^{(q)}(f) := \inf_{T_n \in \mathbb{T}_n \cap \Delta^{(q)}(Y)} \|f - T_n\|, \quad n \in \mathbb{N},$$

where $\mathbb{T}_n$ is the space of trigonometric polynomials of order $\le n-1$.

Theorem 1 *If $f \in \Delta^{(q)}(Y)$ then*

$$E_n^{(q)}(f) \le c(s)\omega_k(f, \pi/n), \quad n \ge N(Y), \quad k = \begin{cases} 2, & \text{if } q = 1, \\ 3, & \text{if } q = 0, 2, \end{cases}$$

where $\omega_k(f,t)$ is the k-th modulus of continuity of f, $c(s)$ and $N(Y)$ are the constants depending only on s and on $\min_{i=1,\dots,2s}\{y_i - y_{i+1}\}$, respectively.

Remark 1 Each of these three estimates is wrong with a greater k. It follows from the Whitney inequality that the constants $c(s)$ and $N(Y)$ can be both replaced simultaneously by $c(Y)$ and 1, respectively. The respective estimates with $c(s)$ and 1 are wrong.

The case $q = 0$ was proved by the author and J. Gilewicz, $q = 1$ by the author and M. G. Pleshakov, $q = 2$ by the pupil of the author V. D. Zalizko.