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A Self-dual Variational Calculus and its Applications

How to solve PDEs by completing squares? Motivated in part by the basic equations of quantum field theory (e.g. Yang–Mills, Ginzburg–Landau, etc.. . .), we identify a large class of partial differential equations that can be solved via a newly devised "self-dual" variational calculus.

In both stationary and dynamic cases, such self-dual equations are not derived from the fact they are critical points of action functionals, but because they are also zeroes of appropriately chosen non-negative Lagrangians. The class contains many of the basic families of linear and nonlinear, stationary and evolutionary partial differential equations: Transport equations, Nonlinear Laplace equations, Cauchy–Riemann systems, Navier–Stokes equations, but also infinite dimensional gradient flows of convex potentials (e.g. heat equations), nonlinear Schrödinger equations, Hamiltonian systems, and many other parabolic-elliptic equations.