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The Birman–Craggs–Johnson homomorphism and the cohomology of the Torelli group

The Torelli group T_g is the subgroup of the mapping class group of a surface consisting of those diffeomorphism classes acting trivially on the homology of the surface. The Torelli group is the “mysterious part” of the mapping class group, and basic questions about its structure (e.g., finite presentability) are still not known; in particular its homology is known only in dimension one. In the 1970/80s Birman–Craggs–Johnson constructed a remarkable surjective homomorphism from T_g to a certain $\mathbb{Z}/2\mathbb{Z}$ -vector space of Boolean (square-free) polynomials. The heart of this construction comes from the Rochlin invariant for homology 3-spheres. This homomorphism, together with abelian cycles in T_g , can be used to construct nontrivial elements of $H_2(T_g, \mathbb{Z}/2\mathbb{Z})$, which had not been detected rationally. This construction yields a lower bound on the order of g^4 for the rank of this homology group. We will also discuss the question of lifting these classes to integral classes and the connection with the “Casson–Morita algebra”.

This is joint work in progress with Benson Farb.