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The 4th Doctoral Prize
Le 4^{ième} prix de doctorat

Citation

In his thesis Steve studies properties of Cantor sets and their application to problems of Diophantine approximation. The thickness of a Cantor set is a measure of the size of the set and, according to a result of Newhouse from 1970, the sum of two Cantor sets is an interval if the product of their thicknesses is at least one. Steve has been able to generalize Newhouse's theorem to the sum of any number of Cantor sets. He has also established a lower bound for the thickness of the sum of two Cantor sets in terms of the thickness of each set and proved that, in general, the lower bound is best possible. Such a result had been sought for at least 30 years.

In a famous paper from 1947 Marshall Hall proved that every real number can be expressed as a sum of two badly approximable numbers. In particular, he showed that every real number can be expressed as a sum of two numbers which, when expanded as continued fractions, have partial quotients at most 4. Let us denote this set of such numbers by $F(4)$ and define $F(m)$ correspondingly for each positive integer m . In 1973 Divis showed that one could not replace 4 by 3 in Hall's result. Two years later Hlavka generalized the work of Divis and Hall to study $F(m) + F(n)$ where m and n are positive integers. He proved that

$$F(m) + F(n) = \mathbb{R}, \quad (1)$$

when (m,n) is $(2,7)$ or $(3,4)$ and that

$$F(m) + F(n) \neq \mathbb{R}, \quad (2)$$

when (m,n) is $(2,4)$. This left open the cases $(2,5)$ and $(2,6)$ which Hlavka conjectured satisfied (2). Progress was blocked for 25 years until Steve proved that (1) holds with $(m,n)=(2,5)$, thereby disproving the conjecture of Hlavka and resolving the problem. Let B_1 and B_2 be sets of positive integers. Steve also established a general criterion for determining when every real number can be represented as a sum of two real numbers with one of the numbers having partial quotients from B_1 and the other having them from B_2 . A key element in the proof of these results is his work on the thickness of Cantor sets.

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Biographical Information

Born and raised in Truro, Nova Scotia, Steve attended Acadia University for his undergraduate degree. He then moved on to the University of Waterloo where he obtained his masters and doctoral degrees under the supervision of Cameron Stewart. While an undergraduate Steve held a Canada Scholarship and won the Governor General's award. During his graduate studies he received funding from NSERC in the form of a post-graduate scholarship, and at present he is an NSERC postdoctoral fellow at the University of Georgia.

The CMS Doctoral Prize was inaugurated to recognize outstanding performance by a doctoral student who graduated from a Canadian University in the preceding year (January 1st to December 31st). The CMS Doctoral Prize consists of an award of \$500, a two-year complimentary membership in the CMS, a framed Doctoral Prize Certificate and a stipend for travel expenses to attend the CMS meeting to receive the award and present a plenary lecture. The first award was presented in 1997.

La SMC a créé le Prix de doctorat pour récompenser le travail exceptionnel d'un étudiant au doctorat en mathématiques ayant obtenu un diplôme d'une université canadienne entre le 1er janvier et le 31 décembre de l'année précédente. Le lauréat du Prix de doctorat de la SMC aura droit à une bourse de 500 \$. De plus, la SMC lui offrira l'adhésion gratuite à la Société pendant deux ans et lui remettra un certificat encadré et une subvention pour frais de déplacements lui permettant d'assister à la réunion de la SMC où il recevra son prix et présentera une conférence. Ce prix a été décerné pour la première fois en 1997.

Recipients / Récipiendaires

1997
1998
1999
2000

James Geelen
Yuri Berest
Jian Shen
Steven Astels