LOUIS P. SAUMIER, University of Victoria

An Efficient Numerical Algorithm for the L2 Optimal Transport Problem with Periodic Densities

We present a numerical method to solve the optimal transport problem with a quadratic cost when the source and target measures are periodic probability densities in any space dimension. This method relies on a numerical resolution of the corresponding Monge-Ampère equation and uses an existing Newton-like algorithm (introduced by Loeper and Rapetti in 2005) that is generalized to the case of a non uniform final density. The main idea consists of designing an iterative scheme where the fully nonlinear equation is approximated by a non-constant coefficient linear elliptic PDE that is discretized and solved at each iteration, in two different ways: a second order finite difference scheme and a fast Fourier transform (FFT) method. The FFT method, made possible thanks to a preconditioning step based on the coefficient-averaged equation, results in an overall O(P log P)-operations algorithm, where P is the number of discretization points. In particular, we employ fourth order finite differences to approximate the action of the densities on the solution iterates, which result in more accurate results without having to sacrifice the efficiency of the overall algorithm. We will state a result showing that the algorithm converges to the solution of the optimal transport problem, under suitable conditions on the densities, and then give the key ideas justifying it. Numerical experiments demonstrating the robustness and efficiency of the method on examples of image processing, including an application to multiple sclerosis (MS) disease detection, will also be presented. This is joint work with Martial Agueh and Boualem Khouider.