JONATHAN BORWEIN, University of Newcastle, NSW

Why Convex?

This lecture makes the case for the study of convex functions focussing on their structural properties. We highlight the centrality of convexity and give a selection of salient examples and applications.

It has been said that most of number theory devolves to the Cauchy–Schwarz inequality and the only problem is deciding 'what to Cauchy with'. In like fashion, much mathematics is tamed once one has found the right convex 'Green's function'. Why convex? Well, because...

References

D. Vanderwerff. Functions: [1] J. Μ. Borwein and J. Convex Constructions. Characterizations and Counterexamples. Encyclopedia Math. Appl. vol. 109. Cambridge University Press. 2009 (http://projects.cs.dal.ca/ddrive/ConvexFunctions/).

The Fourier algebra and the group von Neumann algebra

Let R be the group of real numbers with addition and the usual topology. Then the Banach algebra $L^1(R)$ of integrable functions with convolution product can be identified via the Fourier transform with the Fourier algebra A(R) which is a dense subalgebra of the algebra of continuous functions on R vanishing at infinity with pointwise multiplication.

In this talk, I will introduce the Fourier algebra A(G) of a locally compact group G (not necessarily abelian) which is an algebra of continuous functions on G vanishing at infinity. It can be identified with the predual of the group von Neumann algebra of G generated by left translations on $L^2(G)$. Both the Fourier algebra and the group von Neumann algebra play a central role in harmonic analysis on non-abelian groups. In this talk, I will discuss the geometry of A(G) and the Fourier Stieltjes algebra B(G), the associated non-commutative function spaces in the group von Neumann algebra and some open problems.

NAOMI LEONARD, Princeton University, Princeton, NJ, USA *Stability and Robustness of Collective Dynamics*

Given the joint challenge to explain the enabling mechanisms of collective behavior in social animal groups and to define provable mechanisms of collective behavior for networked robotic groups, it is of great value to develop systematic means to study stability and robustness of collective dynamics for multi-agent systems. When distributed feedback laws used by individual agents depend only on measurements of relative states of others, the closed-loop dynamics retain a symmetry, and synchrony measures can be used to parametrize solutions in shape space. I will discuss stability of synchronized behaviors and robustness of synchrony to input heterogeneity, as a function of the (directed) inter-agent sensing topology. Dispersion, which measures the distance from synchrony, is examined for networked dynamical systems in the presence of external input disturbances with bounded L_2 norm. Robustness is formalized with an L_2 gain condition and the dependence is derived of the L_2 gain on the sensing topology and on properties of the individual agent dynamics. Other robustness measures are considered for classes of systems.

ANTHONY TO-MING LAU, Department of Mathematical and Statistical Sciences, University of Alberta, Edmonton, Alberta, T6G 2G1

NANCY REID, University of Toronto *Likelihood inference in complex models*

The likelihood function underpins nearly all statistical inference and modelling, but with very complex models the likelihood function can be cumbersome or impossible to compute. Various simplifications have been suggested for particular settings, and recently the method of composite likelihood has been widely used. This method, which uses only lower dimensional distributions instead of the full joint distribution to construct the model, seems to have good efficiency as well as ease of calculation. I will discuss some models and applications where various versions of composite likelihood seem to perform well, with a view to understanding the reasons for this good performance, and also understanding when it may give misleading results.

CHRISTINE SHOEMAKER, Cornell University, Hollister Hall, Ithaca, NY 14850, USA

Continuous Optimization with Response Surfaces for Computationally Expensive Simulation Models Including Environmental Applications

This talk will present an overview of algorithms that employ response surfaces to significantly reduce the computational effort required to solve continuous optimization and uncertainty analysis of nonlinear simulation models that require a substantial amount of CPU time for each simulation. For nonlinear objective functions and simulation models, the resulting optimization problem is usually multimodal and hence requires a global optimization method.

In order to reduce the number of simulations required, we are interested in utilizing information from all previous simulations done as part of an optimization search by building a (radial basis function) multivariate response surface that interpolates these earlier simulations. I will discuss the alternative approaches of direct global optimization search versus using a multistart method in combination with a local optimization method. These different approaches will be illustrated by two global optimization response surface methods to come from our group recently. I will also briefly describe an uncertainty analysis method SOARS that uses derivative-free optimization to help construct a response surface. This approach has been shown to reduce CPU requirements to less than 1/65 of what is required by conventional MCMC uncertainty analysis. I will present examples of the application of these methods to significant environmental problems described by computationally intensive simulation models used worldwide. One model (MODFLOW/MT3D) involves partial differential equation models for groundwater and the second is SWAT, which is used to describe potential pollution of NYC's drinking water. In both cases, the model is applied to data from a specific site.

DAVID VOGAN, MIT, Cambridge, Massachusetts, USA *Signatures of Hermitian forms and unitary representations*

Suppose G is compact Lie group. The representations of G—possible ways of realizing G as group of matrices—provide a powerful way to organize the investigation of a wide variety of problems involving symmetry under G. For example, if G acts by isometries on a Riemannian manifold, each eigenspace of the Laplace operator is a representation of G. Knowing the possible dimensions of representations can therefore tell you about possible multiplicities of Laplacian eigenvalues.

When G is noncompact, there may be no realizations of G using finite matrices, and those involving arbitrary infinite matrices are too general to be useful. Stone, von Neumann, Wigner, and Gelfand realized in the 1930s that unitary operators on Hilbert spaces provided a happy medium: that any group could be realized by unitary operators, but that the possible realizations could still be controlled in interesting examples.

Gelfand's "unitary dual problem" asks for a list of all the realizations of a given group G as unitary operators. Work of Harish-Chandra, Langlands, and Knapp–Zuckerman before 1980 produced a slightly longer list: all realizations of G as linear operators preserving a possibly indefinite Hermitian form. I will describe a notion of "signatures" for such infinite-dimensional forms, and recent work of Jeff Adams' research group "Atlas of Lie groups and representations" on an algorithm for calculating signatures. This algorithm identifies unitary representations among Hermitian ones, and so promises to resolve the unitary dual problem.