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The n-body problem in spaces of constant curvature

We generalize the Newtonian n-body problem to spaces of curvature $k =$ constant, and study the motion in the 2-dimensional case. For $k > 0$, the equations of motion encounter non-collision singularities, which occur when two bodies are antipodal. This phenomenon leads, on one hand, to hybrid solution singularities for as few as 3 bodies, whose corresponding orbits end up in a collision-antipodal configuration in finite time; on the other hand, it produces non-singularity collisions, characterized by finite velocities and forces at the collision instant. We also point out the existence of several classes of relative equilibria, including the hyperbolic rotations for $k < 0$. In the end, we prove Saari's conjecture when the bodies are on a geodesic that rotates elliptically or hyperbolically. We also emphasize that fixed points are specific to the case $k > 0$, hyperbolic relative equilibria to $k < 0$, and Lagrangian orbits of arbitrary masses to $k = 0$—results that provide new criteria towards understanding the large-scale geometry of the physical space.