Plenary Speakers
Conférenciers pléniers

MARCELO C. BORBA, UNESP–São Paulo State University
Modeling, Projects and Internet: Alternatives to undergraduate basic mathematics courses

Since the late 1970s, applied mathematicians brought to Brazilian mathematics education the notion that working with "real problems" could be an alternative to mathematics education at different levels. Specifically, some aspects of mathematical modeling were introduced into undergraduate calculus courses offered to non-math majors. Some mathematics educators have transformed this idea and have encouraged students to choose themes for project work. This approach became known in Brazil as modeling and has changed curriculum since the teacher and the institution alone no longer choose all the contents and topics to be studied. Over the last fifteen years, I have developed research using such an approach in basic calculus courses proffered to Biology majors at UNESP. In this talk, I will discuss the educational underpinnings of such an approach and present results from research about the kind of mathematics that emerge (or not) from such projects. I will also show how the availability of the Internet and computer technology in general has shaped the work with such a pedagogical approach.

ERICH KALTOFEN, North Carolina State University
Expressing a Fraction of Two Determinants as a Determinant

Suppose the polynomials f and g in $\mathsf{K}[x_1, \ldots, x_r]$ over the field K are determinants of $m \times m$ and $n \times n$ matrices, respectively, whose entries are in $\mathsf{K} \cup \{x_1, \ldots, x_r\}$. Furthermore, suppose $h = f/g$ is a polynomial in $\mathsf{K}[x_1, \ldots, x_r]$ and suppose that K has at least $m + 1$ elements. We construct an $s \times s$ matrix C whose entries are in $\mathsf{K} \cup \{x_1, \ldots, x_r\}$, such that $h = \det(C)$ and $s = O\big((m + n)^6\big)$. Our problem was motivated by resulant formulas derived from Chow forms.

Additionally, we show that divisions can be removed from formulas that compute polynomials in the input variables over a sufficiently large field within polynomial formula size growth.

This is joint work with Pascal Koiran at the ENS Lyon, France.

MIKHAIL KAPRANOV, Yale University, Mathematics Dept., New Haven, CT 06520, USA
Algebro-geometric models for the spaces of unparametrized paths

In several applications such as string theory, one has to deal with the spaces of paths or loops in a manifold X defined up to reparametrization. Such quotienting by reparametrization is usually quite difficult to perform. On the other hand, the holonomy of any connection along a path is obviously reparametrization invariant. This allows us to define an algebro-geometric object Π_X which can be considered as the formal neighborhood of X (punctual paths) in the space (groupoid) of reparametrization equivalence classes of paths in X. This is done by a version of Tannakian duality for the tensor category of vector bundles on X with not necessarily flat connections. In particular, $\mathcal{P}_X$, the Lie algebroid of the groupoid Π_X is the universal receptacle of "higher covariant derivatives of the curvature" of various connections. It can be considered as a noncommutative analog of the sheaf of vector fields. When X is a smooth projective variety, we establish a relation of Π_X with the moduli stack of stable curves of Kontsevich.

GIOVANNI LANDI, University of Trieste, via A. Valerio 12/1, I-34127 Trieste
Dirac operators on noncommutative manifolds

I will motivate the use of noncommutative geometry for singular spaces and give an introduction to the use of Dirac operators (and of spectral triples) for metric structures on noncommutative manifolds. The general theory will be clarified by some crucial examples coming from toric and quantum group (co)-actions.

BLAINE LAWSON, SUNY Stony Brook, Stony Brook, NY 11794
Projective Hulls, Projective Linking, and Boundaries of Analytic Varieties

This talk revolves generally on the question: When does a real closed curve γ in a complex manifold X bound a holomorphic surface $\Sigma \subset X$? In a classic paper in 1959 John Wermer gave a beautiful answer for the case $X = \mathbf{C}^n$. He showed that γ bounds a one-dimensional subvariety $\Sigma \subset \mathbf{C}^n$ if and only if the restrictions of complex polynomials to γ are not dense in the continuous functions. In fact, let A denote the closure of this algebra of polynomials in $C(\gamma)$. Then Wermer showed that Σ can be identified with the Gelfand spectrum of A.

In a different direction, H. Alexander and J. Wermer later considered oriented curves $\gamma \subset \mathbf{C}^n$ (not necessarily connected) and asked when does γ form the boundary of a positive holomorphic 1-chain (a positive integral linear combination of 1-dimensional subvarieties). They proved that this happens if and only if γ has non-negative linking with all algebraic subvarieties in $\mathbf{C}^n - \gamma$.

I shall discuss analogues of these results in complex projective space and more generally in projective algebraic manifolds. This will entail the notion of the projective hull of a compact subset of projective space and the concept of projective linking numbers. The results extend to higher dimensional submanifolds $M \subset X$. They lead to an interesting duality result concerning the classes represented by positive holomorphic chains. This duality asserts that under the pairing $H_{2p}(X, M : \mathbf{Q}) \times H_{2n-2p}(X_M\,; \mathbf{Q}) \to \mathbf{Q}$, the classes in each group represented by positive holomorphic chains are polars of each other.

SETH LLOYD, Massachusetts Institute of Technology, MIT 3-160, Cambridge, MA 02139, USA
Quantize clocks, not gravity

Time is not an observable in quantum gravity. What, then, do clocks measure? If you pose this question to researchers at the National Institute of Standards and Technology (NIST), they will tell you, 'Our clocks do not measure time. Our clocks define time.' That is, time is defined by the observed phases of quantum clocks. Similarly, in quantum field theory, space is not an observable. Distances in space are derived, e.g., from the observables associated with times of departure and arrival of signals in the global positioning system (GPS). This talk proposes a quantum theory of gravity based on GPS observables.

In this theory, intervals of time and distances in space are defined by clocks and signals. The spacetime metric is not quantized in itself: rather, it obtains its quantum nature from the quantum nature of those clocks and signals.

Quantum fluctuations in times of departure and arrival give rise to quantum fluctuations in the spacetime metric. The 'quantized-GPS' theory of gravity predicts the absence of fluctuations from source-free quantized gravitons in the early universe, an effect that might be observed in the next generation of cosmic microwave background satellites.

OTMAR VENJAKOB, Universität Heidelberg, Mathematisches Institut, INF 288, Zimmer 217, D-69120 Heidelberg, Germany
Are zeta-functions able to solve Diophantine equations?

Motivated by the question whether (some) Diophantine equations are related to special values of ζ- or L-functions, we first describe the origin of classical Iwasawa theory. Then we give a survey on generalisations of these ideas to non-commutative Iwasawa theory, a topic which has been developed in recent years by Burns, Coates, Fukaya, Howson, Huber, Kato, Kings, Ritter, Schneider, Sujatha, Weiss, . . . , and the author.