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Inequalities, equalities between spatial and temporal entropies of cellular automata

A one-dimensional cellular automaton F is a dynamical system on a shift space X that can be defined by a local rule of radius r. For a F and shift invariant measure μ, the temporal entropy $h_\mu(F)$ depends on the way the automaton "moves" the spatial entropy $h_\mu(\sigma)$. Using the discrete average Lyapunov exponents I_μ^+ and I_μ^- we obtain for a shift ergodic and F-invariant measure the inequality:

$$h_\mu(F) \leq h_\mu(\sigma) \times (I_\mu^+ + I_\mu^-).$$

The exponents I_μ^- and I_μ^+ represent the left-to-right and right-to-left average speeds of the faster perturbations. Taking in account the average speed of all the perturbations, we obtain two other equalities:

$$h_\mu(F) = h_\mu(\sigma) \times \int_X \lim_{n\to\infty} M_n(I)(x) \times \frac{I_n(x)}{n} \mathsf{d}\mu(x)$$

and $h_\mu(F) = h_\mu(\sigma) \times M(r) \times r$. The function $M_n(I)(x)$ represents for each x the proportion of perturbations which propagate of $I_n^+(x) + I_n^-(x) = I_n(x)$ coordinates in n iterations. The study of different examples shows that these equalities and inequalities are good tools to show that the entropy of a cellular automaton is equal to zero.

We have also some similar results for the D-dimensional cellular automata $(D > 1)$.