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Categories as Monoids in Span, Rel, and Sup

We look at various internal constructions in various categories and bicategories, that are equivalent to categories. In particular, categories can be expressed as special types of monoids in the category $\mathcal{S}pan$, whose objects are sets, and whose morphisms are spans of functions. In fact, these monoids also live in the category $\mathcal{R}el$, of sets and relations. There is a well-known equivalence between $\mathcal{R}el$, and a full subcategory of the category $\mathcal{S}up$, of complete lattices and sup-preserving morphisms. This allows us to represent categories as a special kind of monoid in $\mathcal{S}up$. Monoids in $\mathcal{S}up$ are called quantales, and are of interest in a number of different areas.

We will also study the appropriate ways to express other categorical structures such as functors, natural transformations and profunctors in these categories.

Joint work with R. Paré.