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The Glueing Construction and Double Categories

For a small category $\mathbf{B}$ and a double category $\mathbb{D}$, let $\mathrm{Lax}_N(\mathbf{B}, \mathbb{D})$ denote the category whose objects are vertical normal lax functors $\mathbf{B} \to \mathbb{D}$ and morphisms are horizontal lax transformations. If $\mathbb{D}$ is the double category of toposes, locales, or topological spaces (see table below), then the glueing construction induces a functor from $\mathrm{Lax}_N(\mathbf{B}, \mathbb{D})$ to the horizontal category $\mathbf{H}\mathbb{D}$.

Objects	Horizontal 1-Cells	Vertical 1-Cells	2-Cells
toposes $\mathcal{X}$	geometric morphisms $\mathcal{X} \to \mathcal{B}$	finite limit preserving $\mathcal{X}_1 \nrightarrow \mathcal{X}_2$	$\mathcal{X}_1 \to \mathcal{B}_1$ $\ddagger \ \leftarrow \ \ddagger$ $\mathcal{X}_2 \to \mathcal{B}_2$
locales X	locale morphisms $X \to B$	finite meet preserving $X_1 \nrightarrow X_2$	$X_1 \to B_1$ $\ddagger \ \geq \ \ddagger$ $X_2 \to B_2$
spaces X	continuous functions $X \to B$	finite meet preserving $\mathcal{O}(X_1) \nrightarrow \mathcal{O}(X_2)$	$\mathcal{O}(X_1) \to \mathcal{O}(B_1)$ $\ddagger \ \geq \ \ddagger$ $\mathcal{O}(X_2) \to \mathcal{O}(B_2)$

For each of these double categories, we know that $\mathrm{Lax}_N(\mathbf{2}, \mathbb{D})$ is equivalent to $\mathbf{H}\mathbb{D}/S$, where $\mathbf{2}$ is the 2-element totally ordered set and S is the Sierpinski object of $\mathbb{D}$. In this talk, we consider analogues of this equivalence for more general categories $\mathbf{B}$.