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Tensor Products of Sup Lattices

The category sup of complete lattices and sup-preserving functions is well known to underlie a tensor category for which the tensor product classifies functions of two variables that preserve suprema in each variable separately. It is classical, after the work of Joyal and Tierney, that sup as a tensor category is somewhat similar to the tensor category of abelian groups. In particular, although much older still, $L \otimes M$ is the quotient of the free sup-lattice $\mathcal{P}(|L| \times |M|)$ by the smallest congruence $\sim$ for which $(\bigvee_i l_i, m) \sim \bigvee_i (l_i, m)$ and $(l, \bigvee_i m_i) \sim \bigvee_i (l, m_i)$.

The arrow $\mathcal{P}(|L| \times |M|) \longrightarrow L \otimes M$ in sup has a fully faithful right adjoint so that it is natural to look for a description of $L \otimes M$ as a full reflective subobject of $\mathcal{P}(|L| \times |M|)$ in ord, the 2-category of ordered sets, order-preserving functions, and inequalities. In fact, it is even more natural to pursue such an approach if we replace the category set of sets and the adjunction $\mathcal{P} \dashv |-| \colon \mathrm{sup} \longrightarrow \mathrm{set}$ by the adjunction $\mathcal{D} \dashv |-| \colon \mathrm{sup} \longrightarrow \mathrm{ord}$. The talk explores this possibility and its extension to tensor products for algebras of KZ-doctrines more generally.

Joint work with Toby Kenney.