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An elementary proof that commutative semigroups satisfy the strong Folner condition

Let $(S,\cdot)$ be a semigroup and let $\mathcal{P}_f(S)$ be the set of finite nonempty subsets of S. Then S satisfies the *Følner condition* (FC) if and only if

$$\big(\forall F\in\mathcal{P}_f(S)\big)(\forall\epsilon>0)\big(\exists K\in\mathcal{P}_f(S)\big)(\forall s\in F)(|sK\setminus K|<\epsilon\cdot|K|).$$

Also S satisfies the *strong Følner condition* (SFC) if and only if

$$\big(\forall F\in\mathcal{P}_f(S)\big)(\forall\epsilon>0)\big(\exists K\in\mathcal{P}_f(S)\big)(\forall s\in F)(|K\setminus sK|<\epsilon\cdot|K|).$$

Any semigroup satisfying SFC has a natural notion of density which is intimately related with the algebraic structure of the Stone–Čech compactification βS of S.

Følner showed that any amenable group satisfies FC and Frey showed that any left amenable semigroups satisfies FC. Argabright and Wilde showed that any semigroup satisfying SFC is left amenable. They also showed that any commutative semigroup satisfies SFC. However their proof of the latter fact relied on the fact that any commutative semigroup is amenable, a fact whose proof is not particularly easy and is a bit hard to find, as well as the fact above that any amenable semigroup satisfies FC. We provide a simple elementarly (and entirely algebraic) proof that any commutative semigroup satisfies SFC.