Groups and Hopf Algebras
Groupes et algèbres de Hopf

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ELI ALJADEFF, Technion (Haifa), Israel
Specht Problem for G-graded Algebras

Let W be an algebra over a field F of characteristic zero. Let $\mathrm{id}(W)$ be the T-ideal of identities of W, i.e., polynomials in noncommutative variables which vanish upon any evaluation on W. One of the main theorems in PI theory (due to Kemer, 1991) is the solution of the Specht problem, namely that $\mathrm{id}(W)$ is finitely generated as a T-ideal (an ideal of the free algebra $F\langle X\rangle$ is a T-ideal if it is closed under endomorphisms, that is, variables can be replaced by arbitrary polynomials). The main problem in the solution is the case where the algebra W is affine (i.e., finitely generated over F).

In the last two decades people considered G-graded identities on G-graded algebras where G is any group (generalizing ordinary identities where $G=\{1\}$). Here one considers polynomials in noncommutative G-graded variables, that is variables of the form x_g where $g \in G$. Admissible evaluations are those where the variables x_g are replaced only by elements of W_g. A G-graded identity is a polynomial which vanishes upon any admissible evaluation. The Specht problem for G-graded algebras asks whether the T-ideal of G-graded identities is finitely generated as a T-ideal. The case where $G \cong \mathbb{Z}/2\mathbb{Z}$ was known (due to Kemer) and it was essential in reducing the ordinary Specht problem from non-affine to affine algebras.

In these lectures I will present the main steps in the proof of the Specht problem for PI, G-graded affine algebras where G is a finite group (the nonaffine case follows as in the non-graded case). I will explain the obstacles which arise from the fact that the group G may not be abelian. As in the ungraded case the main part is to show that the T-ideal of G-graded identities of an affine algebra W coincides with the T-ideal of G-graded identities of a G-graded finite dimensional algebra A.

It should be emphasized that a G-graded algebra may be G-graded PI (i.e., PI as a G-graded algebra) even if it is non-PI (e.g., take the free algebra W (on more than one variable), G-graded where $G \neq \{e\}$ and $W_g = 0$ for $g \neq e$). The Specht problem remains open for such algebras.

Most of the new results which I will present were obtained jointly with Alexei Kanel Belov. Other results were obtained with Haile, Natapov, Kassel, Giambruno and La Mattina.

ANDREAS BÄCHLE, University of Stuttgart, Germany
On torsion units in $\mathbb{Z}\,\mathrm{Sz}(q)$

Let $V(\mathbb{Z}G)$ denote the group of normalized units of the integral group ring $\mathbb{Z}G$, i.e., the units with coefficient sum 1. A more than 30-year-old conjecture of Hans Zassenhaus states:

(ZC1) For a finite group G every torsion unit u in $V(\mathbb{Z}G)$ is conjugate within $\mathbb{Q}G$ to an element of G.

This conjecture has been verified for some classes of groups, but only for very few non-solvable groups. Luthar, Passi and Hertweck developed a method to deal with this conjecture with arithmetical means using the characters of the groups. With the aid of this tool the following for the smallest group of the series of the simple Suzuki groups $\mathrm{Sz}(q)$ is shown: The orders of torsion units in $V\big(\mathbb{Z}\,\mathrm{Sz}(8)\big)$ coincide with orders of elements of $\mathrm{Sz}(8)$; if the order is 2, 5 or 13 the elements are conjugate by units of $\mathbb{Q}\,\mathrm{Sz}(q)$ to group elements. This gives an affirmative answer to a question of Kimmerle, namely if the prime graphs of $V(\mathbb{Z}G)$ and G coincide, in this case.

Recently Hertweck, Höfert and Kimmerle showed that for all prime powers q and all primes r the finite r-subgroups of $V\big(\mathbb{Z}\,\mathrm{PSL}(2\,;q)\big)$ are isomorphic to subgroups of $\mathrm{PSL}(2\,;q)$. Using similar methods and the above result we obtained that for a finite minimal simple group G and a prime r any elementary abelian r-group $H \leq V(\mathbb{Z}G)$ is isomorphic to a subgroup of G, except possibly the case $G \cong \mathrm{PSL}(3\,;3)$ and $H \cong C_3 \times C_3 \times C_3$.

FRAUKE BLEHER, University of Iowa

Universal deformation rings and group rings

Let k be a perfect field of positive characteristic p, and let G be a finite group. There are various classical results in the literature concerning lifts of finitely generated kG-modules to complete discrete valuation rings of characteristic 0, such as Green's liftability theorem. To understand and generalize these results, it is useful to reformulate them in terms of deformation rings. Deformation rings have become an important tool in the area of number theory, and in particular in the area of Galois representations and modular forms.

In this talk, I will introduce universal deformation rings and deformations of finitely generated kG-modules. This definition goes back to B. Mazur who used deformation rings to study lifts of Galois representations. I will show how the knowledge of the structure of the group ring of kG and its blocks together with the structure of the corresponding blocks in characteristic 0 can be used to determine the universal deformation rings of certain indecomposable kG-modules V. In particular, I will give examples when the universal deformation ring of V is directly related to the group ring of a defect group of the block to which V belongs.

STEFAAN CAENEPEEL, Vrije Universiteit Brussel

Cohomology of commutative Hopf algebroids

In the first part, we discuss restricted Picard groupoid; these can be viewed as the two-dimensional version of abelian groups. Every restricted Picard groupoid is monoidal equivalent to a strict Picard groupoid, this is a Picard groupoid in which the symmetry, evaluation and coevaluation are the identity morphisms. For strict Picard groupoids, we have a structure theorem. We can define complexes of strict Picard groupoids, and the corresponding cohomology groups. To such a complex, two other complexes of abelian groups can be associated, and the cohomology groups of the three complexes are connected by a long exact sequence of cohomology, generalizing, as we will show later, the long exact sequence of Villamayor and Zelinsky. We introduce cosimplicial Picard groupoids, and associate a complex of Picard groupoids to it.

Hopf algebroids can be viewed as the proper generalization of Hopf algebras to non-commutative base rings. We will consider commutative Hopf algebroids over commutative rings; these are still more general than commutative Hopf algebras: the main difference is that we have a left and right unit map; in the case where these coincide, we recover the classical definition of commutative Hopf algebra. Other examples include: the Sweedler canonical coring associated to a commutative ring extension; $A \otimes_R H$, where H is a commutative Hopf algebra over R, and A a commutative right H-comodule algebra; this example can be generalized to the case where H is a weak Hopf algebra.

To a commutative Hopf algebroid $\mathcal{A}$, we can associate a cosimplicial commutative ring. Given a covariant functor to abelian groups or to restricted Picard groupoids, we can then construct a cosimplicial abelian group or restricted Picard groupoid. We can then consider the corresponding cohomology groups. For this covariant functor, we choose $\underline{\mathrm{Pic}}$, associating the group of invertible S-modules to a commutative ring S. The corresponding cohomology groups are then denoted $H^n(\mathcal{A}, \underline{\mathrm{Pic}})$. The above exact sequence takes the form

$$\begin{aligned} 1 &\longrightarrow H^1(\mathcal{A}, \mathbb{G}_m) \xrightarrow{\alpha_1} H^0(\mathcal{A}, \underline{\mathrm{Pic}}) \xrightarrow{\beta_1} H^0(\mathcal{A}, \mathrm{Pic}) \\ &\xrightarrow{\gamma_1} H^2(\mathcal{A}, \mathbb{G}_m) \xrightarrow{\alpha_2} H^1(\mathcal{A}, \underline{\mathrm{Pic}}) \xrightarrow{\beta_2} H^1(\mathcal{A}, \mathrm{Pic}) \\ &\xrightarrow{\gamma_2} \cdots . \end{aligned}$$

In the case where $\mathcal{A} = S \otimes_R S$, the Sweedler canonical coring, the cohomology that we obtain is Amitsur cohomology, and the above exact sequence is the Villamayor–Zelinsky exact sequence; in the case where $\mathcal{A} = H$ is a commutative Hopf algebra, we recover Harrison cohomology, and, in the case where H is finitely generated and projective, Sweedler cohomology over H^*.

We give algebraic interpretations of the cohomology groups $H^n(\mathcal{A}, \underline{\mathrm{Pic}})$ in the case where $n = 0, 1, 2$. $H^0(\mathcal{A}, \underline{\mathrm{Pic}})$ is isomorphic to the Picard group of invertible $\mathcal{A}$-module corings. To describe $H^1(\mathcal{A}, \underline{\mathrm{Pic}})$, we have to describe a Galois theory of $\mathcal{A}$-module corings. In the case where $\mathcal{A} = S \otimes_R S$, the duals of the Galois coobjects are Azumaya algebras split by S. In the case where $\mathcal{A} = H$, the duals of the Galois coobjects are H-Galois objects in the classical sense. Finally, $H^2(\mathcal{A}, \underline{\mathrm{Pic}})$ classifies certain monoidal structures on the category of $\mathcal{A}$-modules. As an application of our theory, we obtain a normal basis theorem for Galois coobjects over commutative weak Hopf algebras.

Joint work with B. Femić.

JUAN CUADRA, Universidad de Almeria, Dpto. Algebra y Analisis Matematico, 04120 Almeria, Spain
Computing the Brauer group of certain quasi-triangular Hopf algebras

In this talk we will attain a deeper understanding of recent computations of the Brauer group of some quasi-triangular Hopf algebras by explaining why a direct product decomposition for this group holds and describing one of the factors occurring in it. For a Hopf algebra B in a braided monoidal category $\mathcal{C}$, and under certain assumptions on the braiding (fulfilled if $\mathcal{C}$ is symmetric), we will show that:

(1) The Brauer group $\mathrm{BM}(\mathcal{C}\,;B)$ of B-module algebras is isomorphic to $\mathrm{Br}(\mathcal{C}) \times \mathrm{Gal}(\mathcal{C}\,;B)$, where $\mathrm{Br}(\mathcal{C})$ is the Brauer group of $\mathcal{C}$ and $\mathrm{Gal}(\mathcal{C}\,;B)$ the group of B-Galois objects;

(2) $\mathrm{BM}(\mathcal{C}\,;B)$ contains a subgroup isomorphic to $\mathrm{Br}(\mathcal{C}) \times \mathrm{H}^2(\mathcal{C}\,;B, I)$, where $\mathrm{H}^2(\mathcal{C}\,;B, I)$ is the second Sweedler cohomology group of B with values in the unit object I of $\mathcal{C}$.

These results will be applied to the Brauer group of a quasi-triangular Hopf algebra that is a Radford biproduct $B \times H$, where H is a usual Hopf algebra over a field, the Hopf subalgebra generated by the quasi-triangular structure $\mathcal{R}$ is contained in H and B is a Hopf algebra in the category ${}_H\mathcal{M}$ of left H-modules.

The results presented in this talk are part of a joint work with Bojana Femić (arXiv:0809.2517).

FLORIAN EISELE, University of Aachen, Germany
Algorithms for p-adic group rings

Let $(k, \mathcal{O}, K)$ be a p-modular system. Some algorithms are presented that allow calculations in the module category of an $\mathcal{O}$-order Λ in a separable K-algebra A. In particular the methods admit calculation of the projective indecomposable Λ-lattices as amalgams of irreducible lattices. An application is the calculation of basic algebras of a group ring $\mathcal{O}G$, provided the irreducible ordinary representations of G are known. The algorithms have been implemented using the computer algebra system GAP.

GASTON GARCÍA, National University of Cordoba, Argentina
Quantum Subgroups of $\mathrm{GL}_{\alpha,\beta}(n)$

Let $\alpha, \beta \in \mathbb{C} \setminus \{0\}$ and $\ell \in \mathbb{N}$, $\ell \geq 3$. We determine all Hopf algebra quotients of the quantized coordinate algebra $\mathrm{Oc}_{\alpha,\beta}(\mathrm{GL}_n)$ when $\alpha^{-1}\beta$ is a primitive ℓ-th root of unity and α, β satisfy certain mild conditions, and we characterize all finite-dimensional quotients when $\alpha^{-1}\beta$ is not a root of unity. As a byproduct we give a new family of non-semisimple and non-pointed Hopf algebras with non-pointed duals which are quotients of $\mathrm{Oc}_{\alpha,\beta}(\mathrm{GL}_n)$.

This problem was first considered by P. Podleś for one parameter deformations of $\mathrm{SU}(2)$ and $\mathrm{SO}(3)$. Then, the characterization of all finite-dimensional Hopf algebra quotients of the one-parameter deformation $\mathcal{O}_q(\mathrm{SL}_N)$ of the coordinate algebra of SL_N was obtained by Eric Müller, and in a joint work with N. Andruskiewitsch, *Quantum subgroups of a simple quantum group at roots of 1* (to appear in Compositio Mathematica), we determined all Hopf algebra quotients of the quantized coordinate algebra $\mathcal{O}_q(G)$, where G is a connected, simply connected simple complex algebraic group and q is a primitive ℓ-th root of 1. In order to determine the Hopf algebra quotients of the two-parameter deformation of $\mathcal{O}(\mathrm{GL}_n)$ we need to work both with our approach and Müller's approach, which is via explicit computations with matrix coefficients.

ALLEN HERMAN, University of Regina
The Rings of Generalized Groups

“Generalized Groups” (Hecke algebras and non-commutative association schemes being two examples of interest) are algebraic structures that share many of the algebraic and representation-theoretic properties of finite groups, and some useful additional properties as well. Rather than try give a survey of what has become a vast subject, I will present a few illustrative examples.

MARTIN HERTWECK, University of Stuttgart, Germany
Units in integral group rings and Zassenhaus conjectures

[no abstract text supplied]

ERIC JESPERS, VUB, Belgium

LARS KADISON, University of Pennsylvania
Depth of a Subgroup

A subalgebra pair of semisimple complex algebras $B \subset A$ with inclusion matrix M is depth two if $MM^tM < nM$ for some positive integer n and all corresponding entries. If A and B are the group algebras of finite group-subgroup pair $H < G$, the induction-restriction table for irreducible characters equals M, and $S = MM^t$ satisfies $S^2 < nS$ iff the subgroup H is depth three in G; similarly depth $n > 3$ by successive right multiplications of this inequality with alternately M and M^t. For example, the pair of permutation groups $S_n < S_{n+1}$ has depth $2n - 1$ (or more). In joint work with Kuelshammer and Burciu, we show that a subgroup H has depth $2n + 2$ if its core is an intersection of H with n conjugates of H.

YEVGENIA KASHINA, DePaul University
Applications of Frobenius–Schur indicators

Given an irreducible character of a finite group, one can define its (second) Frobenius–Schur indicator by evaluating this character on the sum of the squares of all elements of the group, divided by the order of the group. Linchenko and Montgomery extended this definition to semisimple Hopf algebras and generalized the classical Frobenius–Schur theorem from groups to semisimple Hopf algebras. The set of Frobenius–Schur indicators proved to be a very important invariant of a Hopf algebra; in particular they can be used to compute the trace of the andipode of a Hopf algebra. In this talk we will discuss some recent applications of Frobenius–Schur indicators.

CHRISTIAN KASSEL, University of Strasbourg & CNRS, France
Invariant Drinfeld twists on group algebras

Drinfeld twists were introduced by Drinfeld in his work on quasi-Hopf algebras; they have been used to classify certain classes of (co)semisimple Hopf algebras. In joint work with Pierre Guillot (arXiv:0903.2807), after observing that the invariant Drinfeld twists on a Hopf algebra form a group, we determine this group when the Hopf algebra is the algebra of a finite group G. The answer involves the group of class-preserving outer automorphisms of G as well as all abelian normal subgroups of G of central type. In my lecture I shall also present several examples for which the group of invariant twists has been completely computed by us.

WOLFGANG KIMMERLE, University of Stuttgart, Germany
On group rings of finite simple groups

It is well known that a finite simple group G is determined up to isomorphism by its ordinary character table and therefore by its integral group ring. But probably much weaker conditions suffice. The object of the talk are recent results of M. Nagl, M. Borgart and myself related to the following two conjectures.

Conjecture (Huppert 2000). Let G be a finite simple group and let H be a finite group. Assume that the set of the irreducible ordinary character degrees of G coïncides with that one of H. Then $H \cong G \times A$ with A abelian.

Note that the assumption in Huppert's conjecture is satisfied provided G and H have isomorphic complex group algebras. More general for an arbitrary field K the question whether a finite (almost) simple group G is determined by the group algebra KG will be considered. Huppert's conjecture is from the point of view of character tables dual to the following one.

Conjecture (Thompson 1988). Let G be a finite simple group and let H be a finite group with trivial centre. Assume that the set of the conjugacy class lengths of G concides with that one of H. Then $G \cong H$.

ALEXANDER KONOVALOV, University of St. Andrews, Scotland
On Orders of Torsion Units in Integral Group Rings of Sporadic Simple Groups

Let $V(\mathbb{Z}G)$ be the normalized unit group of the integral group ring $\mathbb{Z}G$ of a finite group G. The long-standing conjecture of H. Zassenhaus (ZC) says that every torsion unit from $V(\mathbb{Z}G)$ is conjugate within the rational group algebra $\mathbb{Q}G$ to an element of G.

W. Kimmerle proposed to relate (ZC) with some properties of graphs associated with groups. The Gruenberg–Kegel graph (or the prime graph) of the group G is the graph with vertices labelled by the prime divisors of the order of G with an edge from p to q if and only if there is an element of order pq in the group G. Then Kimmerle's conjecture (KC) asks whether G and $V(\mathbb{Z}G)$ have the same prime graph.

We started the project of verifying (KC) for sporadic simple groups, using the Luthar–Passi method with recent extensions by M. Hertweck as a main tool. Now we are already able to report that (KC) holds for the following thirteen sporadic simple groups:

- Mathieu groups $M_{11}, M_{12}, M_{22}, M_{23}, M_{24}$;
- Janko groups J_1, J_2, J_3;
- Held, Higman–Sims, McLaughlin, Rudvalis and Suzuki groups.

In my talk I will summarise known information about orders and partial augmentation of these groups, explain enhancements of the Luthar–Passi method that were developed during the project, and highlight some challenges arising from the remaining sporadic simple groups.

Joint work with Victor Bovdi, Eric Jespers, Steve Linton, Salvatore Siciliano et al.

MATTHIAS KUENZER, University of Aachen, Germany
A Fourier–Hopf inversion and spectral sequences

Let H be a Hopf algebra over a commutative ring R. Let K be a normal Hopf subalgebra of H. Write $\bar{H} := H/HK^+$. Suppose given an H-module M. Then $\operatorname{Hom}_K(H, M)$ and $\operatorname{Hom}_R(\bar{H}, M)$ are isomorphic as left $\bar{H}$-modules, the isomorphism resembling a bit a Fourier inversion. Under some projectivity assumptions, this can be used to show that $\operatorname{Hom}_K(H, M)$ is $\operatorname{Hom}_{\bar{H}}(R, -)$-acyclic. Therefore, in this situation there are two spectral sequences,

(1) the Grothendieck spectral sequence, working with injective Cartan–Eilenberg resolutions, and

(2) a more pedestrian one, working with a tensor product of projective resolutions.

Moreover, (1) and (2) are isomorphic, so in particular, they converge both to $\operatorname{Ext}^*_H(R, M)$. This specialises to groups (Lyndon–Hochschild–Serre) and to Lie algebras (Hochschild–Serre).

AARON LAUVE, Texas A&M University, Dept. of Mathematics, MS 3368, College Station, TX 77843-3368, USA
Nichols algebras in positive characteristic

Nichols algebras were introduced by Nichols in 1978 and have reappeared in several places since (including work on the Schubert calculus and quantum enveloping algebras). More recently, they appear as key players in the classification of pointed Hopf algebras (which are finite dimensional if and only if their Nichols subalgebras are). The goal of this talk is to advertise one particular realization of Nichols algebras based on the Hopf quivers of Cibils and Rosso. It is hoped that this model will eventually be useful in determining large families of Nichols algebras which are finite dimensional. Here, we present a new family of such algebras in positive characteristic.

Joint work with S. Witherspoon and C. Cibils.

MITJA MASTNAK, Saint Mary's University

Graded bialgebras and their deformations

Various deformation techniques are proving to be instrumental in the efforts to construct new examples of bialgebras as well as in the classification efforts. In the talk I will discuss formal graded deformations, liftings, and cocycle deformations. Emphasis will be placed on the relationships between these concepts.

JASON MCGRAW, Memorial University

Gradings on the Cartan type Lie algebras

We will describe all gradings of the Witt, Special and Hamiltonian algebras by finite groups whose order is coprime to the characteristic of the field. The restriction to groups whose order is coprime to the characteristic allows us to use the correspondence between the grading group and a subgroup of the automorphisms of the algebra in question. We will discuss some gradings of the Contact algebras which are closely related to the gradings on the Hamiltonian algebras.

GABI NEBE, University of Aachen, Germany

p-adic integral group rings

This talk surveys some methods to obtain an explicit description of the Morita equivalence class of a p-adic integral group ring RG of a finite group G which have been introduced by W. Plesken. They are essentially based on linear algebra.

In small cases, the ordinary character table together with the p-modular decomposition matrix of G allow to derive RG by purely combinatorial means. One main ingredient is the fact that RG is a selfdual order with an explicitly given involution. The symmetric groups provide particularly nice examples and we use the Jantzen–Schaper formula to obtain $\mathbf{Z}_p S_{2p}$.

DMITRY NIKSHYCH, University of New Hampshire

Burnside's $p^n q^m$ theorem for Hopf algebras

Let p and q be primes and let H be a semisimple (quasi-)Hopf algebra of dimension $p^n q^m$. We will show that the representation category of H can be obtained from cyclic groups by a sequence of equivariantizations and extensions . This can be viewed as a categorical analogue of the classical Burnside's theorem saying that finite groups of order $p^n q^m$ are solvable. As a consequence we obtain that H contains non-trivial group-like elements and dimensions of irreducible H-modules divide dimension of H, i.e., Kaplansky's 6th conjecture holds for H.

This is a report on a joint work with P. Etingof and V. Ostrik.

JAN OKNINSKI, University of Warsaw, Poland

Algebras, groups and monoids determined by set-theoretic solutions of the Yang–Baxter equation

Set-theoretic solutions of the Yang–Baxter equation lead to fascinating classes of finitely presented algebras, groups and monoids. All of them are determined by a presentation of the form $\langle x_1, \ldots, x_n \mid R \rangle$, where R is a set of $n(n-1)/2$ relations, each of the form $x_i x_j = x_k x_m$. The purpose of the talk is to present the main known results on such algebras, groups and monoids, exhibiting both their structural and combinatorial properties.

Recent solutions of the fundamental problems concerning set theoretic solutions, obtained in a joint work with F. Cedo and E. Jespers, will be also presented.

CESAR POLCINO MILIES, University of Sao Paulo, Brazil
Anticommutativity of symmetric and skew-symmetric elements in group rings

Let RG denote the group ring of a group G over a commutative associative ring with identity R. An involution $g \mapsto g^*$ on G extends linearly to an involution of RG and we shall consider involutions on RG of this type. The symmetric elements of RG under one such involution form a subring of RG if and only if they commute. Necessary and sufficient conditions for this to happen have been studied by several authors. We shall consider an analogous problem: when does $(RG)^-$, the set of skew-symmetric elements, form a subring? It is easy to see that this happens if and only if skew symmetrics anticommute, so we shall discuss this question. The set $(RG)^-$ is closed under the Lie product $[\alpha, \beta] = \alpha\beta - \beta\alpha$ and the problem of deciding when this product is trivial has also received a lot of attention. We shall discuss an analogous question: when is the Jordan product trivial in the set of symmetric elements; i.e., when do the symmetrics anticommute?

Joint work with Edgar Goodaire.

DAVID RADFORD, University of Illinois, Chicago
Complete Reducibility Theorems for Generalized Quantum Enveloping algebras

Let k be an algebraically closed field of characteristic 0. Recent classification results for certain large classes of pointed Hopf algebras by Andruskiewitsch and Schneider show that generalizations of quantized enveloping algebras and the small quantum groups of Lusztig cover quite a bit of ground.

We discuss a generalization of the complete reducibility theorem for the quantized enveloping algebras and other complete reducibility theorems for them. In some cases arguments follow those in Lusztig's book very closely.

This is nearly completed work of Andruskiewitsch, Radford, and Schneider.

SERBAN RAIANU, California State University, Dominguez Hills
The antipode of a co-Frobenius dual quasi-Hopf algebra is bijective

For a co-Frobenius Hopf algebra of arbitrary dimension over a field, it is well-known that the space of integrals is one-dimensional and the antipode is bijective. Daniel Bulacu and Stefaan Caenepeel recently showed that for a dual quasi-Hopf algebra with nonzero integrals the space of integrals is one-dimensional, and the antipode is injective. We prove that the antipode is bijective.

This is joint work with Margaret Beattie and Miodrag Iovanov.

BAHRAM RANGIPOUR, UNB Fredericton
Hopf algebras of transverse transverse symmetries, characteristic map and its generalizations

In this talked we give a short review of the recent development of Hopf algebras arising from transverse geometries. We mostly emphasis on their algebraic construction and its application in Hopf cyclic cohomology. Then a new class of coefficients for this cohomology will be introduced and finally a cup product based on this coefficients will be defined.

HANS-JÜRGEN SCHNEIDER, University of Munich, Germany
Hopf algebras and root systems

The first fundamental problem in the classification theory of pointed Hopf algebras is to understand Nichols algebras (or quantum symmetric algebras). Prominent examples of Nichols algebras are the $+$-parts of the quantum groups of semisimple Lie algebras. More concretely a very general and basic question is the following: What is the structure of the Nichols algebra of a finite direct sum V of finite-dimensional irreducible Yetter–Drinfeld modules (over any Hopf algebra with bijective antipode)? It turns out that there is a rich combinatorial context which can be used to answer this question.

In joint work with I. Heckenberger we associate a generalized root system to a Nichols algebra of this type (like the root system of a Kac–Moody Lie algebra), and in recent joint work with N. Andruskiewitsch and I. Heckenberger we define reflections of a Weyl groupoid of the Nichols algebra of V (like the Weyl group of a generalized Cartan matrix). I will give an introduction to these new developments and to some of their applications.

A. C. SOUZA-FILHO, Universidade de São Paulo
Free Groups in Quaternion Algebras

Hyperbolic groups were first studied by M. Gromov. The Flat Plane Theorem states that if a group is hyperbolic it has no subgroup isomorphic to $\mathbb{Z}\times\mathbb{Z}$. This result has an important role in the classification of the groups G and the rational quadratic extensions $K=\mathbb{Q}\sqrt{-d}$ such that the group of normalized units $V=\mathcal{U}_1(\mathfrak{o}_K[G])$ is a hyperbolic group, where $\mathfrak{o}_K$ is the integral algebraic ring of K. If G is a non-abelian group, Juriaans, Passi and Souza Filho proved that V is hyperbolic if, and only if, G is the quaternion group of order 8 and d is a positive square free integer such that $d\equiv 7 \pmod 8$. In this case, the quaternion algebra $\mathcal{H}(K)$ is a division ring. Since the unit group of the $\mathbb{Z}$-order $\Gamma=\mathcal{H}(\mathfrak{o}_K)$ is hyperbolic, we also show that for a suitable pair of units u, v of Γ there is an integer m such that the group generated by the powers u^m, v^m is a free subgroup of Γ of rank two. Juriaans and Souza Filho extended this result and show that, in fact, the theorem is true for any positive square free integer d. We also prove that the power m is equal to 1, except in the case $d=2$ which $m=2$.

In this talk we shall communicate results obtained by S. O. Juriaans and A. C. Souza Filho in which the units u, v are constructed by the methods due to S. O. Juriaans, I. B. S. Passi and A. C. Souza Filho.