Symplectic and Contact Geometry
Géométrie symplectique et de contact
(Org: **Emmanuel Giroux** (CNRS-ENS Lyon) and/et **Yael Karshon** (Toronto))

DENIS AUROUX, MIT, Department of Mathematics, 77 Massachusetts Ave., Cambridge, MA 02139, USA
Special Lagrangian fibrations and mirror symmetry

This talk will focus on a geometric proposal for constructing the mirror of a compact Kähler manifold equipped with an anticanonical divisor, extending the Strominger–Yau–Zaslow conjecture beyond the Calabi–Yau case. The mirror manifold is constructed as a (complexified) moduli space of special Lagrangian tori, and the Landau–Ginzburg superpotential is defined by a weighted count of holomorphic discs. We will give examples, both in the toric and in the non-toric setting, to illustrate the construction and the manner in which instanton corrections arise from exceptional discs and wall-crossing phenomena.

JEAN-FRANÇOIS BARRAUD, Université Lille 1, 59655 Villeneuve d'Ascq Cédex, France
The path loop fibration in symplectic topology

The well known Morse homology uses the critical points of a Morse function and 0-dimensional spaces of flow lines to recover the homology of the ambient manifold. I will explain how this complex can be enriched to take higher dimensional moduli spaces into account. As expected, the resulting algebraic invariant is a refinement over homology, and consists in the Leray–Serre spectral sequence associated to the path-loop fibration. This construction turns out to have interesting applications and ramifications in the symplectic setting, that will be discussed as well.

PATRICK BERNARD, Université Paris-Dauphine
Mather Theory, Hamilton–Jacobi equation, and symplectic geometry

We shall give a presentation of Mather theory where the invariant sets are defined as intersections between a given hypersurface and certain Lagrangian manifolds of phase space.

PAUL BIRAN, Tel-Aviv University
A Floer–Gysin long exact sequence for Lagrangian submanifolds

In this talk we will present a Floer-homological version of the classical Gysin sequence for Lagrangian circle bundles. The Floer homology analogue of this sequence looks apparently similar: the singular chomology groups are replaced by Floer cohomologies and the Euler class by an appropriate quantum version of it which we call the Floer–Euler class. However, for many Lagrangians the algebraic properties of this exact sequence are quite different than the classical one.

We will present several applications of this technique to computations of Floer cohomologies, to the study of the topology of Lagrangian submanifolds and to questions concerning rational curves on Fano varieties.

Joint work with Michael Khanevsky.

BASAK GÜREL, Centre de Recherches Mathematiques (CRM), Montreal, Canada
Leaf-wise coisotropic intersections

The Lagrangian intersection property is unquestionably one of the most fundamental results in symplectic topology. Namely, a Lagrangian submanifold necessarily intersects its image under a Hamiltonian diffeomorphism that is in some sense close to the identity.

It is natural to consider generalizations of the Lagrangian intersection property to coisotropic submanifolds. Among several different versions of the coisotropic intersection property is the question of leaf-wise intersections. In this talk we will discuss our recent work on this problem which is also connected to some problems in geometric mechanics and mathematical physics.

MEGUMI HARADA, McMaster University

Hyperkähler Kirwan surjectivity for quiver varieties: Morse theory and examples

Let G be a compact Lie group. The well-known Kirwan surjectivity theorem in equivariant symplectic geometry states that the G-equivariant rational cohomology of a Hamiltonian G-space (M,ω) surjects onto the ordinary rational cohomology of the symplectic quotient of M by G. This surjective ring homomorphism ("the Kirwan map") has been a key tool in computations of the topology of symplectic quotients.

I will discuss our recent progress on the analogous hyperkähler question, namely: if $(M,\omega_1,\omega_2,\omega_3)$ is a hyperkähler hyperhamiltonian G-space, then does the G-equivariant cohomology of M surject onto the ordinary rational cohomology of the hyperkähler quotient of M by G? We restrict to the case of Nakajima quiver varieties and develop a Morse theory for the hyperkähler moment map analogous to the case of the moduli space of Higgs bundles. In particular, we show that the Harder–Narasimhan stratification of spaces of representations of quivers coincide with the Morse-theoretic stratification associated to the norm-square of the real moment map.

Our approach also provides insight into the topology of specific examples of small-rank quiver varieties, including hyperpolygon spaces and some ADHM quivers.

This is a preliminary report on work in progress with Graeme Wilkin.

SAMUEL LISI, Stanford University, Stanford (California)

Compactness and non-compactness for generalized pseudoholomorphic curves

The study of pseudoholomorphic curves in symplectizations of contact manifolds has given many new results in the study of Reeb dynamics, over the course of the last 15 years. A large number of remarkable results in 3-manifolds come from studying foliations of the symplectization by pseudoholomorphic punctured spheres (for instance, Hofer, Wysocki and Zehnder proved that for a Baire set of contact forms, tight S^3 has either two or infinitely many periodic orbits). For index and intersection reasons, it is useful to consider a more general class of curves, if the genus is positive. These generalized pseudoholomorphic curves satisfy a Cauchy–Riemann type equation twisted by a harmonic one form. While the local theory remains the same as for standard pseudoholomorphic curves, the Gromov-type compactness results no longer hold.

This talk will present joint work with Abbas and Hofer on understanding the compactness properties of these curves, and will give examples of ways in which these curves behave quite differently from standard pseudoholomorphic curves.

KLAUS NIEDERKRÜGER, Ecole Normale Supérieure de Lyon, UMR CNRS 5669, 46 allée d'Italie, 69364 Lyon Cedex 07, France

Negative stabilizations and the plastikstufe

According to E. Giroux, the negative stabilization of a contact open book (of arbitrary dimension) should give an "overtwisted" contact structure. Such manifolds contain an object that resembles a plastikstufe. I will try to explain what needs to be done to prove that this degenerate plastikstufe also implies non-fillability of the contact manifold.

ALEXANDRU OANCEA, Université Louis Pasteur, Institut de Recherche Mathématique Avancée (IRMA), 7 rue Descartes, 67084 Strasbourg, France

Symplectic homology and linearized contact homology

The setup considered is that of an exact symplectic manifold W with contact type boundary M. I will explain that linearized contact homology of M is isomorphic to positive S^1-equivariant symplectic homology of W. This result is part of a bigger picture: any version of symplectic homology is isomorphic to a suitable version of linearized contact homology, and conversely, so that the two theories are entirely parallel. The dominant point of view is that of S^1-equivariant constructions and Gysin exact sequences.

Joint work with Frédéric Bourgeois (Brussels) and Kai Cieliebak (Munich).

MARTIN PINSONNAULT, CRM, Université de Montréal

Symplectic balls in rational ruled 4-manifolds

Many symplectic rigidity phenomena involve either symplectic balls or Lagrangian submanifolds. In this talk, I will explain how the study of integrable complex structures on rational ruled 4-manifolds leads naturally to a homotopy theoretic description of the spaces of embedded symplectic balls. These results, obtained in a joint work with S. Anjos and F. Lalonde, reveal the complexity of these embedding spaces and, conjecturally, should translate into similar complexity results for some spaces of Lagrangian submanifolds.

JEAN-YVES WELSCHINGER, Ecole normale supérieure de Lyon

Sphères lagrangiennes et symétrie miroir homologique

La conjecture de symétrie miroir homologique, proposée par M. Kontsevich, fait intervenir la catégorie de Donaldson–Fukaya des variétés symplectiques à première classe de Chern nulle. Il s'agit d'une catégorie dont les objets sont les sous-variétés lagrangiennes de la variété symplectique. Comme le souligne Kontsevich, cette catégorie n'est cependant pas bien définie en général pour ces variétés, pour la même raison que l'homologie de Floer n'est pas bien définie pour des lagrangiennes dans ces variétés. Je montrerai grâce à des méthodes de théorie symplectique des champs comment bien définir cette catégorie en se restreignant aux sphères lagrangiennes.