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Rough blow up solutions to the L^2 critical NLS

The cubic nonlinear Schrödinger equation in dimension 2

$$i\partial_t u + \Delta u + u|u|^2 = 0$$

is a canonical model for the focusing of a laser beam. The question of the qualitative description of both the long time behavior of the solution and the description of the possible singularity formation has attracted a considerable amount of both formal and rigorous works for the past twenty years, in particular due to the fact that the NLS structure is considered to be canonical for a large number of problems. Following a breakthrough work of Bourgain in the 90's, a challenging problem is in particular to obtain a qualitative description of the flow in the critical space L^2 . I will show how the merging of oscillatory integrals techniques as developed by Colliander, Keel, Staffilani, Takaoka and Tao for the study of the Cauchy problem for rough data, with more elliptic in nature estimates developed by Merle and Raphaël for the study of the singularity formation in the energy space H^1 , allows one to derive a blow up theory in the almost critical space H^s , $\forall s > 0$.

This is joint work with Jim Colliander.