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Philosophical Implications of Recent Work in Set Theory

Cohen's method of forcing allows one to show that many concrete mathematical statements are undecidable from the axioms of ZFC (Zermelo–Fraenkel set theory with the Axiom of Choice). A natural question arises: Can one repair the weaknesses in ZFC exposed by forcing, and if so, to what extent? The investigation of this question involves the study of higher axioms of infinity (the so-called “large cardinal axioms”), their canonical models, and the determinacy of infinite games. Recently, Woodin has developed Ω -logic, a strong extension of first-order logic that is the natural logic given by the method of forcing. He has also developed a transfinite proof system for Ω -logic, and isolated the Ω Conjecture, which is essentially completeness for this logic and its corresponding proof system. A proof of the Ω Conjecture would quantify the limits of forcing, provide a possible solution to the Continuum Hypothesis, show that those large cardinal axioms which admit an inner model theory of the kind that we know today are “cofinal” amongst all large cardinal axioms, and challenge the popular conception that there is no need for additional axioms of set theory.