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The Stability of a General Functional Equation

Suppose that $\mathbb{V}$ is a vector space over $\mathbb{Q}$, $\mathbb{R}$ or $\mathbb{C}$, the scalars $\alpha_0, \beta_0, \ldots, \alpha_m, \beta_m$ are such that $\alpha_j \beta_k - \alpha_k \beta_j \neq 0$ whenever $0 \leq j < k \leq m$, $\mathbb{B}$ is a Banach space, $f_k \colon \mathbb{V} \to \mathbb{B}$ for $0 \leq k \leq m$, $\delta \geq 0$ and

$$\Big\| \sum_{k=0}^{m} f_k(\alpha_k x + \beta_k y) \Big\| \leq \delta \quad \text{for all } x, y \in \mathbb{V}.$$

Then, for each $k = 0, 1, \ldots, m$ there exists $c_k \in \mathbb{B}$ and a "generalized" polynomial function $p_k \colon \mathbb{V} \to \mathbb{B}$ of "degree" at most $m-1$, such that

$$\|f_k(x) - c_k - p_k(x)\| \leq 2^{m+1}\delta \quad \text{for all } x \in \mathbb{V}$$

and

$$\sum_{k=0}^{m} p_k(\alpha_k x + \beta_k y) = 0 \quad \text{for all } x, y \in \mathbb{V}.$$

Moreover, if $\mathbb{V} = \mathbb{R}^n$, $\mathbb{B} = \mathbb{R}$ or $\mathbb{C}$ and, for some j, f_j is bounded on a set of positive Lebesgue measure, then every p_k is a genuine polynomial function.