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Forced Symmetry Breaking from $\mathrm{SO}(3)$ *to* $\mathrm{SO}(2)$ *for Rotating Waves on the Sphere*

Geometrical imperfections (*e.g.*, the shape of a heart is not exactly a sphere) and localized inhomogeneities (*e.g.*, small blood vessels) inherent in the cardiac tissue can influence the dynamics of the spiral waves. In both cases, it can be considered that the spherical symmetry $\mathrm{SO}(3)$ is broken to $\mathrm{SO}(2)$ symmetry.

In this talk, we consider an ϵ-small perturbation of a reaction-diffusion system on the sphere of radius r, which is only $\mathrm{SO}(2)$-equivariant for $\epsilon > 0$, but $\mathrm{SO}(3)$-equivariant for $\epsilon = 0$. The effects of forced symmetry breaking for rotating waves on the sphere of radius r are presented.

Namely, for $\epsilon = 0$, we consider a normally hyperbolic relative equilibrium $\mathrm{SO}(3)u_0$ with trivial isotropy. It persists to an $\mathrm{SO}(2)$-invariant normally hyperbolic flow-invariant manifold $M(\epsilon)$. We study the dynamics on $M(\epsilon)$ using the orbit space reduction methods and Poincaré–Bendixson theorem on the sphere. The problem reduces to the study of the following differential equations on the unit sphere $\mathbf{S}^2$:

$$\dot{x} = -[X_0 + \epsilon g^S(x,\epsilon)]x, \quad \epsilon \geq 0 \text{ small}, \tag{1}$$

where $X_0 \in \mathrm{so}(3)$, $g^S \colon \mathbf{S}^2 \times [0, \epsilon_0) \to \mathrm{so}(3)$. We analyze the differential equations (**??**) using the Implicit function theorem and the Poincaré map.

Then, we obtain that depending on the frequency vectors of the rotating waves that form $\mathrm{SO}(3)u_0$, these rotating waves (up to $\mathrm{SO}(2)$) will give either $\mathrm{SO}(2)$-orbits of rotating waves or $\mathrm{SO}(2)$-orbits of modulated rotating waves (if some transversality conditions hold). The orbital stability of these solutions is established as well.