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Geometric Realizations of Non-Convex Simplicial Spheres

The study of combinatorial types of convex polytopes has been an extremely fruitful one for twentieth-century mathematics. In 1967, Grünbaum and Shreedharan discovered some errors in Brückner's 1909 enumeration of the simple convex polytopes with 8 facets in 4-dimensional Euclidean space. In particular, they discovered that one of the combinatorial types listed by Brückner did not admit a realization as the boundary of a convex polytope. Since then, much work has been done on determining which simplicial spheres admit realizations as the boundaries of convex polytopes; we will call such spheres *convexly realizable*. This talk will review some of that work and explore some of the interesting questions that arise when one begins to investigate the geometric structure of those simplicial spheres that are *not* convexly realizable.