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The modularity of some $\mathrm{mod}\, p$ *Galois representations*

I will sketch the main ideas of recent preprints of Khare–Wintenberger and Dieulefait that allow one to establish certain cases of Serre's conjectures on $\mathrm{mod}\, p$ irreducible two-dimensional representations of the Galois group of Q. Recall that the 1994 proof of Fermat's Last Theorem for exponent p associates an elliptic curve E to each putative non-trivial solution of Fermat's equation. The $\mathrm{mod}\, p$ representation $E[p]$ associated to E is incompatible with Serre's conjecture; it cannot be modular. The classical method of Wiles is to prove that $E[p]$ is modular by proving that E is modular. In the new millennium, Khare–Wintenberger and Dieulefait work directly with $E[p]$ and forget where it came from. While the new method may not strike everyone as a simplification of the proof of FLT, it will very likely lead to striking new results.