QIJI ZHU, Western Michigan University Helly's Intersection Theorem on Manifolds of Nonpositive Curvature

We give a generalization of the classical Helly's theorem on intersection of convex sets in \mathbb{R}^N for the case of manifolds of nonpositive curvature. In particular, we show that if any N + 1 sets from a family of closed convex sets on N-dimensional Cartan–Hadamard manifold contain a common point, and at least one of set is compact then all sets from this family contain a common point. Our proof use a variational argument. In \mathbb{R}^N this proof is rather straightforward yet it seems new to us. The generalization to manifolds of nonpositive curvature relies on tools for nonsmooth analysis on smooth manifolds that we developed recently.

This is joint research with Yuri Ledyaev and Jay Treiman.