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A new type of differential equation arising from economic theory

In optimal control, the discount rate is always exponential, that is, a gain of u occurring at a distance (in time) t from now is worth a gain $u \exp(-rt)$ today, where $r > 0$ is the interest rate. Using this expression, one derives the classical Hamilton–Jacobi–Bellman equation.

In economics, there is no reason to favour exponential discount rates. Much interest recently has been paid to discount rates $h(t)$, where $h(0) = 1$, h is decreasing and $h(t)$ goes to zero when t goes to infinity. With such a discount rate, the optimal control loses economic significance, and must be replaced by an equilibrium strategy. The latter is given by a new equation, which resembles the HJB equation, but which is no longer a PDE.