Algebra
(Org: **R. Buchweitz** (Toronto), **J. de la Peña** (UNAM) and **A. Pianzola** (Alberta))

MICHAEL BAROT, UNAM
Graduated inclusions between simply-laced semi-simple Lie algebras: a description with unit forms

Denote by Δ and Γ two simply-laced *Dynkin types*, that is, disjoint unions of simply-laced Dynkin diagrams, and by $g(\Delta)$ and $g(\Gamma)$ the semi-simple Lie algebras of that type and recall that they are graduated by the root spaces $g(\Delta)_\alpha$.

When does there exist a *graduated* inclusion $\varphi\colon g(\Delta) \hookrightarrow g(\Gamma)$ (here graduated means: there exists a linear map f such that $\varphi\big(g(\Delta)_\alpha\big) \subseteq g(\Gamma)_{f(\alpha)}$)?

We translate this question into the language of *unit forms*, that is, integer quadratic forms $q\colon \mathbb{Z}^n \to \mathbb{Z}$ satisfying $g(c_i) = 1$ for each canonical base vector c_i. This enables us to give a complete answer to the previous question.

The talk will present results from a joint work with José Antonio de la Peña.

RAYMUNDO BAUTISTA, Universidad Nacional Autónoma de México, Unidad Morelia, Apartado Postal 61-3 (Xangari), CP 58089 Morelia, Michoacán, México
Representations of tame algebras over rational functions

In the following we use the following notation. If B is a finite-dimensional algebra over a field F, we denote by $B\text{-mod}$ the category of finitely generated left B-modules. By $F(x)$ we denote the field of rational functions over x. We put $F(x,y) = F(x)(y)$.

Let A be a finite-dimensional algebra over the algebraically closed field k. We put $A^{k(x)} = A \otimes_k k(x)$ and $A^{k(x,y)} = A \otimes_k k(x,y)$.

We prove the following result:

Theorem *The algebra A is of tame representation type if and only if for any indecomposable object M in $A^{k(x,y)}\text{-mod}$ such that ${}_{A^{k(y)}}M$ is an indecomposable $A^{k(y)}$-module, there is an indecomposable object N in $A^{k(x)}\text{-mod}$ with ${}_AN$ an indecomposable A-module such that*

$$M \cong N \otimes_{k(x)} k(x,y).$$

Joint work with Leonardo Salmeron.

YULY BILLIG, Carleton University, Ottawa
Thin coverings of modules

In this talk we will discuss a method of constructing graded-simple modules from ungraded simple modules over graded algebras (associative or Lie). This method works both in finite-dimensional and infinite-dimensional quasifinite set-ups. The key ingredient in our construction is the action of a cyclotomic quantum torus on the module. We apply this method to get a description of irreducible representations for the twisted toroidal Lie algebras.

This is based on a joint work with Michael Lau.

THOMAS BRUESTLE, Bishops and Université de Sherbrooke, Québec, Canada
Cyclic cluster algebras of rank three

Cluster algebras, introduced by Fomin and Zelevinsky a few years ago, have gained a lot of interest by now. Acylic cluster algebras have been shown to be related to cluster categories and tilted algebras. Cyclic cluster algebras, however, are less well understood.

We consider the first non-trivial case, cluster algebras of rank three (square, coefficient-free), and study which of them are cyclic. Rank three cluster algebras are given by triples of integers (x, y, z), and we provide an answer which involves the hyperplanes defined by

$$x^2 + y^2 + z^2 - xyz = c.$$

This is joint work with Ibrahim Assem, Martin Blais and Lutz Hille.

RAGNAR-OLAF BUCHWEITZ, University of Toronto at Scarborough, UTSC, 1265 Military Trail, Toronto, ON, M1C 1A4, Canada

Noncommutative Version of Hochschild Cohomology

It is a celebrated result by Gerstenhaber from 1964 that (classical) Hochschild cohomology is graded commutative for any associative algebra. Suarez–Alvarez's elegant treatment of the categorical Eckmann–Hilton argument yields easily the same property for derived Hochschild cohomology as defined by Quillen. There is a canonical algebra homomorphism from classical to derived Hochschild cohomology that factors through the Yoneda algebra of the self-extensions of the given algebra as a bimodule over itself.

The question addressed here is whether the latter Yoneda algebra might also always be graded commutative. That is known under mild "Tor-transversality" conditions, when that algebra already coincides with the derived version of Hochschild cohomology. Here we give a simple example, the normalization of a plane cusp singularity, where the answer is negative. Indeed, the Yoneda Ext-algebra in that case is essentially an infinitely generated tensor algebra.

VLADIMIR CHERNOUSOV, University of Alberta, Edmonton, Alberta, Canada

Zero cycles on projective homogeneous varieties

We present a new method of computing the Chow group of zero cycles on projective homogeneous varieties which is based on an idea of parametrization of splitting fields.

JOSE ANTONIO DE LA PEÑA, UNAM, México DF

Spectra of Coxeter polynomials

Let A be a finite dimensional algebra over an algebraically closed field k. Assume A has finite global dimension. The Auslander–Reiten translation τ_A defines an automorphism in the derived category of the module category mod_A. The linear transformation induced on the Grothendieck group is called the Coxeter transformation and the associated characteristic polynomial f_A is the Coxeter polynomial of A. The spectra of the Coxeter polynomial is related with important properties of the algebra: the structure of the Auslander–Reiten quiver of A, the growth of the iterated translations $\tau^n[X]$ for indecomposable modules X and other facts. For hereditary algebras $A = kQ$ with Q a quiver, f_A is known to be closely related to the characteristic polynomial of the adjacency matrix of the underlying graph of Q. We study new classes of algebras where the spectra of f_A can be described by means of characteristic polynomials of adjacency matrices of graphs.

IVAN GUTMAN, University of Kragujevac, Faculty of Science, POB 60, 34000 Kragujevac, Serbia

Energy of a Graph

Let G be a graph on n vertices. Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be its eigenvalues (*i.e.*, the eigenvalues of the adjacency matrix of G). The energy of G is defined as [1]

$$E(G) = \sum_{i=1}^{n} |\lambda_i|.$$

The name "energy" was chosen because in certain (limited) cases $E(G)$ is related to the energy of certain molecules. Some fundamental and some newest results on $E(G)$ [2] will be presented, and some open problems indicated.

The quantity

$$EE(G) = \sum_{i=1}^{n} e^{\lambda_i}$$

was recently proposed as a measure of "centrality" of complex networks [3]. Some properties of $EE(G)$ will also be discussed, in particular its relation to $E(G)$.

References

[1] I. Gutman, *The energy of a graph.* Ber. Math.-Statist. Sekt. Forsch. Graz **103**(1978), 1–22.

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[3] E. Estrada and J. A. Rodríguez-Velázquez, *Subgraph centrality in complex networks.* Phys. Rev. **E71**(2005), 056103.

SRIKANTH IYENGAR, Department of Mathematics, University of Nebraska, Lincoln, NE 68588, USA

Hochschild cohomological criteria for the Gorenstein property for commutative algebras

A classical result of Hochschild, Kostant, and Rosenberg characterizes smoothness of commutative algebras essentially of finite type over a field in terms of its Hochschild cohomology. I will discuss a similar characterization of the Gorenstein property.

This is joint work with L. L. Avramov.

GRAHAM LEUSCHKE, Syracuse University, Syracuse, NY 13244, USA

Non-commutative desingularization of the generic determinant

In this joint work with Ragnar-Olaf Buchweitz and Michel Van den Bergh, we show that the hypersurface ring of the generic determinant admits a non-commutative crepant resolution by a "quiverized Clifford algebra".

SHIPING LIU, Université de Sherbrooke, Québec, Canada

The derived category of algebras with radical squared zero

Let A be a finite dimensional elementary algebra over a field with $\mathrm{rad}(A)^0 = 0$. The objective is to study $D^b(A)$, the derived category of bounded complexes in the category of finite dimensional left A-modules. Our technique is to find a proper covering of the ordinary quiver of A so that the complexes of projective A-modules are determined by the representations of the covering. In this way, we are able to give a complete description of the indecomposables, the almost split triangles, the shapes of the components of the Auslander–Reiten quiver of $D^b(A)$ as well as the derived type of A.

This is a joint work with Raymundo Bautista.

ROBERTO MARTINEZ, Instituto de Matemáticas, UNAM, Morelia
On a group graded version of BGG

A major result in Algebraic Geometry is the theorem of Bernstein–Gelfand–Gelfand that states the existence of an equivalence of triangulated categories: $\underline{\mathrm{gr}}_\Lambda \cong \mathcal{D}^b(\mathrm{Coh}\, P^n)$, where $\underline{\mathrm{gr}}_\Lambda$ denotes the stable category of finitely generated graded modules over the $n+1$ exterior algebra and $\mathcal{D}^b(\mathrm{Coh}\, P^n)$ is the derived category of bounded complexes of coherent sheaves on projective space P^n.

Generalizations of this result were obtained in a paper by Martínez-Villa and Saorín and from a different point of view, the theorem has been extended by Yanagawa to Z^n-graded modules over the polynomial algebra. This generalization has important applications in combinatorial commutative algebra.

The aim of the talk is to show how to extend the results to group graded algebras in order to obtain a generalization of Yangawa's results having in mind the application to other settings.

XAVIER GÓMEZ MONT, Centro de Investigación en Matemáticas (CIMAT)
The Homological Index of a Vector Field on an Isolated Complete Intersection Singularity

Given a commutative square of finite free $\mathcal{O}$-modules, we construct a double complex of $\mathcal{O}$-modules, that we have called the Gobelin. (A Gobelin is a richly embroidered French wall tapestry.) The Gobelin is weaved with vertical and horizontal strands of the Buchsbaum–Eisenbud type, constructed each from a Koszul complex of half of the commutative square. We apply the Gobelin to compute the homological index of a germ of a holomorphic vector field on a complete intersection variety, having both an isolated singularity. The first spectral sequence of the Gobelin provides free resolutions of the modules of Kähler differential forms on the complete intersection, and for small degree the homology of the Gobelin coincides with the homology of the complex obtained by contracting differential forms on the complete intersection with the vector field. The second spectral sequence of the Gobelin provides formulas to compute the homology groups of the Gobelin with local linear algebra.

ARTURO PIANZOLA, University of Alberta, Edmonton, Alberta, Canada
Almost commuting subgroups of Lie groups and toroidal Lie algebras

Almost commuting subgroups of Lie groups appear naturally in many areas of Mathematics and Physics (*e.g.* flat connections on tori). The main purpose of this talk is to explain how these subgroups also arise in the Galois cohomology attached to toroidal Lie Algebras.

JUAN RADA, Unversidad de Los Andes
A generalization of the energy to digraphs

The adjacency matrix $A=(a_{ij})$ of a graph G with set of vertices $\{v_1,\ldots,v_n\}$ and set of edges E_G is defined as

$$a_{ij}=\begin{cases}1 & \text{if } v_iv_j\in E_G\\ 0 & \text{if } v_iv_j\notin E_G.\end{cases}$$

The eigenvalues of the graph G are the eigenvalues of the adjacency matrix A. Since A is real and symmetric, the eigenvalues $\lambda_1,\ldots,\lambda_n$ of G are real numbers. The energy of G, denoted by $E(G)$, is defined as

$$E(G)=\sum_{i=1}^{n}|\lambda_i|.$$

One of the long-known results in this field is the Coulson integral formula. In this article, we extend the concept of energy to directed graphs in such a way that Coulson Integral Formula remains valid. As a consequence, it is shown that the energy is increasing over the set $\mathcal{D}_{n,h}$ of digraphs with n vertices and cycles of length h, with respect to a quasi-order relation. Applications to the problem of extremal values for the energy in various classes of digraphs are considered.

FERNANDO SZETCHMAN, Department of Mathematics, University of Regina, Saskatchewan, S4S 0A2, Canada
Irreducible representations of Sylow subgroups of symplectic groups

We construct a canonical family of irreducible representations of a Sylow p-subgroup of the symplectic group $\mathrm{Sp}_{2n}(q)$, where q is a power of an odd prime p. Some of these representations appear in the Weil and Steinberg modules of $\mathrm{Sp}_{2n}(q)$, and a connection between the 2-modular reduction of these will be discussed.

DIETER VOSSIECK, Universidad Michoacana San Nicolás de Hidalgo, Morelia, Michoacán, México
Rigid homomorphisms between finite length modules over a discrete valuation ring

The category $\mathcal{L}_R$ of finite length modules over a discrete valuation ring R is easy to understand: its isomorphism classes correspond bijectively to partitions. However, the category of homomorphisms between finite length R-modules is "wild" and a complete classification of the orbits in $\mathrm{Hom}_R(X,Y)$ under the action of $\mathrm{Aut}_R(X) \times \mathrm{Aut}_R(Y)$ (for all $X, Y \in \mathcal{L}_R$) in terms of normal forms is a hopeless task.

Some time ago we could show that $\mathrm{Hom}_R(X,Y)$ always admits a unique orbit of "rigid" or "generic" homomorphisms. (In the case of the formal power series algebra $R = \mathbf{C}[[T]]$, this means precisely that with respect to the Zariski topology there is a dense open orbit; in the case of the ring of p-adic integers $R = \mathbf{Z}_p$ a similar geometric interpretation can be achieved, using the formalism of Witt vectors.) Moreover we classified the indecomposable rigid homomorphisms, which surprisingly turn out to be certain "strings".

In our talk we will present an algorithm which constructs for given $X, Y \in \mathcal{L}_R$ the essentially unique rigid homomorphism $X \to Y$.