Working Group 10: Mathematics Teacher Training, Algebra, and Teacher Shortages
Atelier 10: Formation des enseignants en mathématiques, algèbre et pénuries d'enseignants

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This working group discussed some of the current issues and challenges in mathematics teacher education at the secondary level. The main questions related to algebra and preservice training, but also touched upon in-service education. More specifically, the three sessions of the working group unfolded as follows.

The first session focused on the notion of big mathematical ideas – those that draw attention to mathematical relationships and offer opportunities for gaining mathematical insight. According to Davey (1999), attention is aesthetic in nature. Whenever we bring consciousness to bear upon a topic, either individually or communally, we engage it in emotional and imaginative ways. We use attention to learn and extend ourselves, to incorporate a new thing, whether it is a new way of solving an old problem or finding new ways to express an idea.

Some of the big ideas in algebra for grades 6-8, as suggested by Edwards (2000), are algebraic notation, variable, function, properties of numbers. Discussion centered on additional candidates for grades 7-12. These included notions of equivalence, the idea that one can manipulate expressions and solve problems by means of algebraic notation without considering the referents for these expressions, and the fact that algebraic symbols can make more visible the structure and properties of number systems.

However, the overall conclusion that emerges from research is that the teaching of algebra is typically instrumental rather than relational, with a dominance of symbolic algebra over other representations (Kieran, 1992; Borba & Confrey, 1996; Kieran & Sfard, 1999). Consequently, though they learn to manipulate algebraic expressions, students do not seem to be able to use them for meaningful mathematical communication. The majority of students do not acquire any real sense of algebra and, early on in their learning of algebra, give up trying to understand algebra and resort to memorizing rules and procedures.

Suggestions from working group members for providing students with meaningful experiences in algebra learning included student exploration of multiple representations of algebraic relationships. It was also suggested that the traditional approach to teaching algebra, which typically starts with symbolic representation and decontextualized manipulation and later moves to graphical representation and problem-based contexts,

should be reversed. An very nice example of an algebra teaching scenario, based on a problem context and using multiple representations to teach, for instance, inequalities, can be found in a Japanese lesson that is available from the set of TIMSS-video case studies (www.intel.com/education/math).

A major question that arises in conjunction with such discussions is how we can help pre-service teachers 'see" the big ideas of algebra and learn to 'live' them in their classrooms. In the search for useful models, the second session was spent studying the case of preservice training in the teaching of mathematics at the secondary level at the Université du Québec à Montréal (UQAM), in particular the two courses of *Didactique de l'algèbre* (Didactics of algebra) and *Didactique de la variable et des fonctions* (Didactics of variable and functions). The role of "didactique" courses is to focus on how to approach the teaching of a specific subject. UQAM has a large group of students specializing in mathematics education. These students are taught by a similarly large group of mathematics education faculty that form a substantial part of the Mathematics Department at UQAM. Working group participants were so intrigued by the UQAM model that they suggested it as the subject of a formal presentation at the next Forum.

The conversation began with a few remarks to situate the UQAM program within the provincial context. Recently, the mathematics curriculum, especially that which concerns the teaching of algebra, has seen major change. Change was imperative in view of the serious difficulties experienced by students in this subject area; the outcome was a new vision of school algebra. Some of the recommendations to be found in the program elaborated for the province of Québec include (see also Bednarz, 2001):

- Less time should be spent in algebraic manipulation;
- The development of algebraic reasoning should be the main focus;
- Algebra should be seen as a means for expressing generality;
- The teaching of algebra ought to be relevant and significant for students.

The content of the UQAM *Didactique de l'algèbre* course was seen by working group participants to include: generalization and rule construction, manipulation of algebraic expressions, equation solving, systems of equations and inequalities, factoring, and the significance of letters in algebra; while that of *Didactique de la variable et des fonctions* involves: study of the main types of functions, translation between representations, roles of parameters, notion of inverse functions, emphasis on verbalization, and a repertoire of situations and their modeling. The discussion of these two didactique courses led to the more general question of the overall structure of the UQAM program in the teaching of mathematics at the secondary level. This four-year program leading to a B.Ed. degree consists of the following courses and practica (all titles have been translated to English and all courses are three credits, unless otherwise indicated): nine mathematics courses (Geometry I and II, Numerical Structures, Linear Algebra and Vector Geometry, Probability and Statistics, Analysis, Equation Theory, Mathematical Explorations with Computers, and History of Mathematics); twelve didactique courses (Didactics of Mathematics I (6 credits) and II (5 credits), Pedagogical Applications of Computing Technology in the Teaching and Learning of Mathematics, Didactics of Algebra, Didactics of Variable and Functions, Proportional Reasoning and Related Concepts,

Didactics of Geometry, Software in the Teaching of Mathematics I and II, Didactics of Mathematical Intervention with Diverse Student Groups, Mathematical and Didactical Perspectives on Various Themes from the Secondary Level, and a Synthesis Seminar (2 credits); twelve courses in Education (33 credits); and four Practica of 2, 5, 5, and 8 credits (all except the first practicum being devoted to the teaching of mathematics).

The third session of the working group involved discussions of other programs of training for mathematics teaching in various Canadian universities. During this session, we compared these programs with the UQAM model and were surprised to learn that a significant number of secondary school mathematics teachers in Quebec were mathematics specialists or had at least some specialized training in the teaching of mathematics. In Ontario, by way of contrast, secondary school teachers start off with two teachable areas, the second requiring only 2-3 university credits. Most of the secondary school teachers teaching mathematics in Ontario are not mathematics specialists, and quite a few of them are not qualified (neither first nor second teachable area).

There is a shortage of teachers having mathematics as either of their teachable subjects, as a result of which many Ontario mathematics classes are given by teachers with little or no university level mathematics. This leads to two related problems for universities – how to encourage more prospective teachers to choose mathematics as an area of specialization at university – and how to enrich the university mathematics background of teachers who do not wish to specialize in mathematics, but who may well end up teaching mathematics courses. The York university experience in providing courses with a low prerequisite threshold, and a rich panorama of mathematical content was discussed. It was pointed out that students would not necessarily take such courses unless they were perceived as helping their admission to education programs.

Recommendations from the working group:

- That CMS partner with teachers associations to publicize looming teacher shortages.
- That data be gathered as to the number and nature of the university mathematics courses taken by mathematics teachers. The data should allow comparison among various grade levels and provinces.
- That universities seek to make courses available to teachers that encourage mathematical thinking, and a broad sense of what mathematics is. These should be available both in pre-service and in-service formats.
- That the 2005 forum include a formal presentation of the UQAM mathematics teacher training model, discussing its unique features and context.
- Slogans: “Supersize your mathematics” (around Big Ideas), “Mathematics specialists for all”.

References:

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