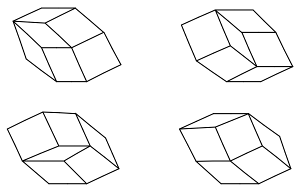
International Mathematical Talent Search – Round 11

Problem 1/11. Express $\dfrac{19}{94}$ in the form $\dfrac{1}{m}+\dfrac{1}{n}$, where m and n are positive integers.

Problem 2/11. Let n be a positive integer greater than 5. Show that at most eight members of the set $\{n+1, n+2, \ldots, n+30\}$ can be primes.

Problem 3/11. A convex $2n$-gon is said to be "rhombic" if all of its sides are of unit length and if its opposite sides are parallel. As exemplified on the right (for the case of $n = 4$), a rhombic $2n$-gon can be dissected into rhombi of sides 1 in several different ways. For what value of n can a rhombic $2n$-gon be dissected into 666 rhombi?



Problem 4/11. Prove that if three of the interior angle bisectors of a quadrilateral intersect at one point, then all four of them must intersect at that point.

Problem 5/11. Let $f(x) = x^4 + 17x^3 + 80x^2 + 203x + 125$. Find the polynomial, $g(x)$, of smallest degree for which $f(3 \pm \sqrt{3}) = g(3 \pm \sqrt{3})$ and $f(5 \pm \sqrt{5}) = g(5 \pm \sqrt{5})$.