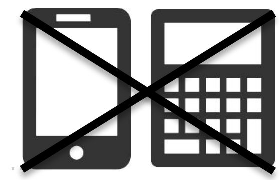
The Sun Life Financial
Canadian Open Mathematics Challenge
November 6/7, 2014

STUDENT INSTRUCTION SHEET

General Instructions



Mobile phones and calculators are NOT permitted.

1) Do not open the exam booklet until instructed to do so by your supervising teacher.

2) The supervisor will give you **five minutes before the exam starts** to fill in the identification section on the exam cover sheet. You don't need to rush. Be sure to fill in all information fields and print legibly.

3) Once you have completed the exam and given it to your supervising teacher you may leave the exam room.

4) The contents of the COMC 2014 exam and your answers and solutions must not be publicly discussed (including online) for at least 24 hours.

Exam Format

You have 2 hours and 30 minutes to complete the COMC. There are three sections to the exam:

PART A: Four introductory questions worth 4 marks each. Partial marks may be awarded for work shown.

PART B: Four more challenging questions worth 6 marks each. Partial marks may be awarded for work shown.

PART C: Four long-form proof problems worth 10 marks each. Complete work must be shown. Partial marks may be awarded.

Diagrams are not drawn to scale; they are intended as aids only.

Work and Answers

All solution work and answers are to be presented in this booklet in the boxes provided – do not include additional sheets. Marks are awarded for completeness and clarity. For sections A and B, it is not necessary to show your work in order to receive full marks. However, if your answer or solution is incorrect, any work that you do and present in this booklet will be considered for partial marks. For section C, you must **show** your work and provide the correct answer or solution to receive full marks.

It is expected that all calculations and answers will be expressed as exact numbers such as 4π, $2 + \sqrt{7}$, etc., rather than as 12.566, 4.646, etc. The names of all award winners will be published on the Canadian Mathematical Society web site https://cms.math.ca/comc.

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Please print clearly and ***complete all information below****. Failure to print legibly or provide complete information may result in your exam being disqualified. This exam is not considered valid unless it is accompanied by your test supervisor's signed form.*

First Name:

Last Name:

Are you currently registered in full-time attendance at an elementary, secondary or Cégep school, or home schooled and have been since September 15th of this year?

☐ Yes ☐ No

Are you a Canadian Citizen or a Permanent Resident of Canada (regardless of current address)?

☐ Yes ☐ No

Grade:

☐ 8 ☐ 9 ☐ 10
☐ 11 ☐ 12 ☐ Cégep
Other: ________

T-Shirt Size:

(Optional. For prize draw)

☐ XS ☐ S ☐ M
☐ L ☐ XL ☐ XXL

Date of Birth:

y y y y m m d d

Gender: (Optional)

☐ Male ☐ Female

E-mail Address:

Signature: ________________________________

For official use only:

A1	A2	A3	A4

B1	B2	B3	B4

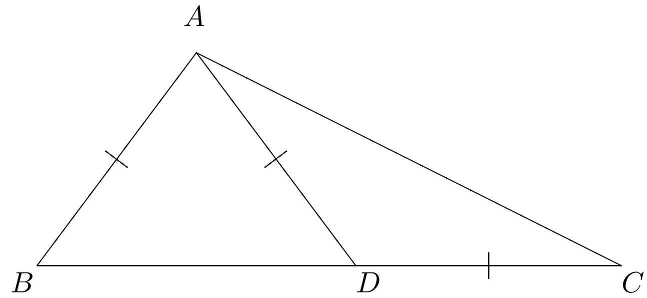
C1	C2	C3	C4

TOTAL

_______ Marker initials _______ Data entry initials _______ Verification initials

Part A: Question 1 (4 marks)

In triangle ABC, there is a point D on side BC such that $BA = AD = DC$. Suppose $\angle BAD = 80°$. Determine the size of $\angle ACB$.



Your Solution:

Your final answer:

Part A: Question 2 (4 marks)

The equations $x^2 - a = 0$ and $3x^4 - 48 = 0$ have the same real solutions. What is the value of a?

Your Solution:

Your final answer:

Part A: Question 3 (4 marks)

A positive integer m has the property that when multiplied by 12, the result is a four-digit number n of the form $20A2$ for some digit A. What is the 4 digit number, n?

Your Solution:

Your final answer:

Part A: Question 4 (4 marks)

Alana, Beatrix, Celine, and Deanna played 6 games of tennis together. In each game, the four of them split into two teams of two and one of the teams won the game. If Alana was on the winning team for 5 games, Beatrix for 2 games, and Celine for 1 game, for how many games was Deanna on the winning team?

Your Solution:

Your final answer:

Part B: Question 1 (6 marks)

The area of the circle that passes through the points $(1, 1), (1, 7)$, and $(9, 1)$ can be expressed as $k\pi$. What is the value of k?

Your Solution:

Your final answer:

Part B: Question 2 (6 marks)

Determine all integer values of n for which $n^2 + 6n + 24$ is a perfect square.

Your Solution:

Your final answer:

Part B: Question 3 (6 marks)

5 Xs and 4 Os are arranged in the below grid such that each number is covered by either an X or an O. There are a total of 126 different ways that the Xs and Os can be placed. Of these 126 ways, how many of them contain a line of 3 Os and no line of 3 Xs?

A line of 3 in a row can be a horizontal line, a vertical line, or one of the diagonal lines $1-5-9$ or $7-5-3$.

1	2	3
4	5	6
7	8	9

Your Solution:

Your final answer:

Part B: Question 4 (6 marks)

Let $f(x) = \dfrac{1}{x^3 + 3x^2 + 2x}$. Determine the smallest positive integer n such that

$$f(1) + f(2) + f(3) + \cdots + f(n) > \frac{503}{2014}.$$

Your solution:

Your final answer:

Part C: Question 1 (10 marks)

A sequence of the form $\{t_1, t_2, ..., t_n\}$ is called *geometric* if $t_1 = a,\ t_2 = ar,\ t_3 = ar^2,\ \ldots\ ,\ t_n = ar^{n-1}$. For example, $\{1, 2, 4, 8, 16\}$ and $\{1, -3, 9, -27\}$ are both geometric sequences. In all three questions below, suppose $\{t_1, t_2, t_3, t_4, t_5\}$ is a geometric sequence of real numbers.

(a) If $t_1 = 3$ and $t_2 = 6$, determine the value of t_5.

(b) If $t_2 = 2$ and $t_4 = 8$, determine all possible values of t_5.

(c) If $t_1 = 32$ and $t_5 = 2$, determine all possible values of t_4.

Your solution:

Part C: Question 2 (10 marks)

The line L given by $5y + (2m - 4)x - 10m = 0$ in the xy-plane intersects the rectangle with vertices $O(0, 0)$, $A(0, 6)$, $B(10, 6)$, $C(10, 0)$ at D on the line segment OA and E on the line segment BC.

(a) Show that $1 \le m \le 3$.

(b) Show that the area of quadrilateral $ADEB$ is $\frac{1}{3}$ the area of rectangle $OABC$.

(c) Determine, in terms of m, the equation of the line parallel to L that intersects OA at F and BC at G so that the quadrilaterals $ADEB$, $DEGF$, $FGCO$ all have the same area.

Your solution:

Part C: Question 3 (10 marks)

A local high school math club has 12 students in it. Each week, 6 of the students go on a field trip.

(a) Jeffrey, a student in the math club, has been on a trip with each other student in the math club. Determine the minimum number of trips that Jeffrey could have gone on.

(b) If each pair of students have been on at least one field trip together, determine the minimum number of field trips that could have happened.

Your solution:

Part C: Question 4 (10 marks)

A polynomial $f(x)$ with real coefficients is said to be a *sum of squares* if there are polynomials $p_1(x), p_2(x), \ldots, p_n(x)$ with real coefficients for which

$$f(x) = p_1^2(x) + p_2^2(x) + \cdots + p_n^2(x)$$

For example, $2x^4 + 6x^2 - 4x + 5$ is a sum of squares because

$$2x^4 + 6x^2 - 4x + 5 = (x^2)^2 + (x^2 + 1)^2 + (2x - 1)^2 + (\sqrt{3})^2$$

(a) Determine all values of a for which $f(x) = x^2 + 4x + a$ is a sum of squares.

(b) Determine all values of a for which $f(x) = x^4 + 2x^3 + (a - 7)x^2 + (4 - 2a)x + a$ is a sum of squares, and for such values of a, write $f(x)$ as a sum of squares.

(c) Suppose $f(x)$ is a sum of squares. Prove there are polynomials $u(x), v(x)$ with real coefficients such that $f(x) = u^2(x) + v^2(x)$.

Your solution:

CMS SMC Canadian Mathematical Society
Société mathématique du Canada

Sun Life Financial
Canadian Open Mathematics Challenge

UNIVERSITY OF TORONTO

UBC

Maplesoft
Mathematics • Modeling • Simulation
A Cybernet Group Company

UNIVERSITY OF CALGARY

DALHOUSIE UNIVERSITY

UNB

UPEI UNIVERSITY of Prince Edward ISLAND

MEMORIAL UNIVERSITY

uOttawa

UNIVERSITY OF MANITOBA

UNIVERSITY OF SASKATCHEWAN

Carleton UNIVERSITY

NELSON EDUCATION