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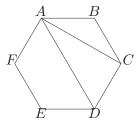
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INSTRUCTIONS: DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED TO DO SO  EXAM: There are 3 parts to the COMC to be completed in 2 hours and 30 minutes.  PART A: Consists of 4 basic questions worth 4 marks each.																				
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Part A: Question 1 (4 marks)	
Determine the positive integer $n$ such that $8^4 = 4^n$ .	
Your Solution:	
Part A: Question 2 (4 marks)	
Tare A. Question 2 (4 marks)	
Let $x$ be the average of the following six numbers: $\{12, 412, 812, 1212, 1612, 2012\}$ . Determine	
the value of $x$ .	
Your Solution:	

#### Part A: Question 3 (4 marks)

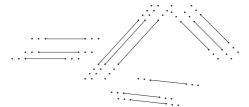
Let ABCDEF be a hexagon all of whose sides are equal in length and all of whose angles are equal. The area of hexagon ABCDEF is exactly r times the area of triangle ACD. Determine the value of r.



**Your Solution:** 

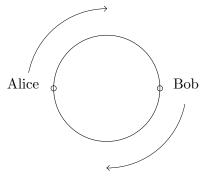
#### Part A: Question 4 (4 marks)

Twelve different lines are drawn on the coordinate plane so that each line is parallel to exactly two other lines. Furthermore, no three lines intersect at a point. Determine the total number of intersection points among the twelve lines.



## Part B: Question 1 (6 marks)

Alice and Bob run in the clockwise direction around a circular track, each running at a constant speed. Alice can complete a lap in t seconds, and Bob can complete a lap in 60 seconds. They start at diametrically-opposite points.



When they meet for the first time, Alice has completed exactly 30 laps. Determine all possible values of t.

Part B: Question 2 (6 marks)
For each positive integer $n$ , define $\varphi(n)$ to be the number of positive divisors of $n$ . For example, $\varphi(10) = 4$ , since 10 has 4 positive divisors, namely $\{1, 2, 5, 10\}$ .
Suppose n is a positive integer such that $\varphi(2n) = 6$ . Determine the minimum possible value of $\varphi(6n)$ .
Your Solution:

### Part B: Question 3 (6 marks)

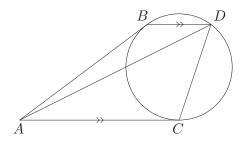
Given the following 4 by 4 square grid of points, determine the number of ways we can label ten different points A, B, C, D, E, F, G, H, I, J such that the lengths of the nine segments

AB, BC, CD, DE, EF, FG, GH, HI, IJ

are in strictly increasing order.

## Part B: Question 4 (6 marks)

In the following diagram, two lines that meet at a point A are tangent to a circle at points B and C. The line parallel to AC passing through B meets the circle again at D. Join the segments CD and AD. Suppose AB = 49 and CD = 28. The length of AD is a positive integer n. Determine n.



# Part C: Question 1 (10 marks)

Let  $f(x) = x^2$  and g(x) = 3x - 8.

- (a) (2 marks) Determine the values of f(2) and g(f(2)).
- (b) (4 marks) Determine all values of x such that f(g(x)) = g(f(x)).
- (c) (4 marks) Let h(x) = 3x r. Determine all values of r such that f(h(2)) = h(f(2)).

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#### Part C: Question 2 (10 marks)

C2 We fill a  $3 \times 3$  grid with 0s and 1s. We score one point for each row, column, and diagonal whose sum is odd.

1	1	0
1	0	1
0	1	1

1	1	1
1	0	1
0	1	1

For example, the grid on the left has 0 points and the grid on the right has 3 points.

(a) (2 marks) Fill in the following grid so that the grid has <u>exactly</u> 1 point. No additional work is required. Many answers are possible. You only need to provide one.

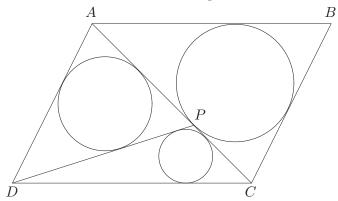


- (b) (4 marks) Determine all grids with exactly 8 points.
- (c) (4 marks) Let E be the number of grids with an even number of points, and O be the number of grids with an odd number of points. Prove that E = O.

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## Part C: Question 3 (10 marks)

Let ABCD be a parallelogram. We draw in the diagonal AC. A circle is drawn inside  $\triangle ABC$  tangent to all three sides and touches side AC at a point P.



- (a) (2 marks) Prove that DA + AP = DC + CP.
- (b) (4 marks) Draw in the line DP. A circle of radius  $r_1$  is drawn inside  $\Delta DAP$  tangent to all three sides. A circle of radius  $r_2$  is drawn inside  $\Delta DCP$  tangent to all three sides. Prove that

$$\frac{r_1}{r_2} = \frac{AP}{PC}.$$

(c) (4 marks) Suppose DA + DC = 3AC and DA = DP. Let  $r_1, r_2$  be the two radii defined in (b). Determine the ratio  $r_1/r_2$ .

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### Part C: Question 4 (10 marks)

For any positive integer n, an n-tuple of positive integers  $(x_1, x_2, \dots, x_n)$  is said to be supersquared if it satisfies both of the following properties:

- (1)  $x_1 > x_2 > x_3 > \cdots > x_n$ .
- (2) The sum  $x_1^2 + x_2^2 + \cdots + x_k^2$  is a perfect square for each  $1 \le k \le n$ .

For example, (12,9,8) is super-squared, since 12 > 9 > 8, and each of  $12^2$ ,  $12^2 + 9^2$ , and  $12^2 + 9^2 + 8^2$  are perfect squares.

- (a) (2 marks) Determine all values of t such that (32, t, 9) is super-squared.
- (b) (2 marks) Determine a super-squared 4-tuple  $(x_1, x_2, x_3, x_4)$  with  $x_1 < 200$ .
- (c) (6 marks) Determine whether there exists a super-squared 2012-tuple.

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# **Canadian Open Mathematics Challenge**

























