The Sun Life Financial
Canadian Open Mathematics Challenge
Wednesday, November 2, 2011

Sun Life Financial

CMS SMC

Please print

TEST SUPERVISOR NAME: ________________________ Signature: __________________________

STUDENT NAME: First: ______________________________ Last: _______________________________

Student Signature: ______________________________ GENDER: ❒ Male ❒ Female

E-MAIL : ____ AGE: ______

GRADE: ❒ 8 ❒ 9 ❒ 10 ❒ 11 ❒ 12 ❒ CEGEP ❒ OTHER: __________

INSTRUCTIONS: DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED TO DO SO

EXAM: There are 3 parts to the COMC to be completed in 2 hours and 30 minutes.

Cell phones and calculators are not permitted.

PART A: Consists of 4 basic questions worth 4 marks each.

PART B: Consists of 4 intermediate questions worth 6 marks each.

PART C: Consists of 4 advanced questions worth 10 marks each.

DIAGRAMS: Diagrams are not drawn to scale; they are intended as aids only.

WORK AND ANSWERS: All solution work and answers are to be presented in this booklet in the space provided. Marks are awarded for completeness and clarity. A correct answer or solution, poorly presented, will not earn full marks. If your answer or solution is incorrect, any work that you do and present in this booklet will be considered for part marks.

It is expected that all calculations and answers will be expressed as exact numbers such as 4π, 2 + , $\sqrt{7}$, etc., rather than as 12:566, 4:646, etc.
The names of all award winners will be published on the Canadian Mathematical Society web site.

The contents of the COMC 2011 and your answers and solutions must not be publically discussed, including web chats, for at least 24 hours.

The Sun Life Financial Canadian Open Mathematics Challenge is a presentation of the Canadian Mathematical Society in partnership with:

Université Laval, University of Toronto, uOttawa, Carleton University, Nelson Education — Canada's Learning Advantage

Do not write in these boxes

A1	A2	A3	A4	A	B1	B2	B3	B4	B	C1	C2	C3	C4	C	ABC

PART A: Question #1 (4 marks)

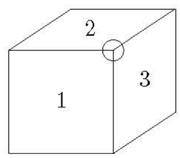
If r is a number such that $r^2 - 6r + 5 = 0$, what is the value of $(r-3)^2$?

PART A: Question #2 (4 marks)

Carmen selects four different numbers from the set {1, 2, 3, 4, 5, 6, 7} whose sum is 11. If ℓ is the *largest* of these four numbers, what is the value of ℓ?

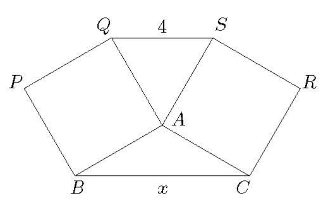
PART A: Question #3 (4 marks)

The faces of a cube contain the number 1, 2, 3, 4, 5, 6 such that the sum of the numbers on each pair of opposite faces is 7. For each of the cube's eight corners, we multiply the three numbers on the faces incident to that corner, and write down its value. (In the diagram, the value of the indicated corner is 1 x 2 x 3 = 6.) What is the sum of the eight values assigned to the cube's corners?



PART A: Question #4 (4 marks)

In the figure, $AQPB$ and $ASRC$ are squares, and AQS is an equilateral triangle. If $QS = 4$ and $BC = x$, what is the value of x?



PART B: Question #1 (6 marks)

Arthur is driving to David's house intending to arrive at a certain time. If he drives at 60 km/h, he will arrive 5 minutes late. If he drives at 90 km/h, he will arrive 5 minutes early. If he drives at n km/h, he will arrive exactly on time. What is the value of n?

PART B: Question #2 (6 marks)

Integers a, b, c, d, and e satisfy the following three properties:

(i) $2 \leq a < b < c < d < e < 100$

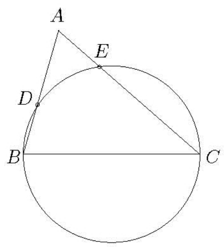
(ii) gcd $(a,e) = 1$

(iii) a, b, c, d, e form a geometric sequence.

What is the value of c?

PART B: Question #3 (6 marks)

In the figure, BC is a diameter of the circle, where $BC = \sqrt{901}$, $BD = 1$, and $DA = 16$. If $EC = x$, what is the value of x?



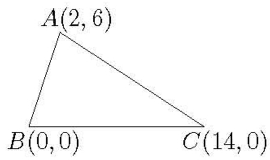
PART B: Question #4 (6 marks)

A group of n friends wrote a math contest consisting of eight short-answer problems $S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8$, and four full-solution problems F_1, F_2, F_3, F_4. Each person in the group correctly solved exactly 11 of the 12 problems. We create an 8 x 4 table. Inside the square located in the i^{th} row and j^{th} column, we write down the number of people who correctly solved both problem S_i and problem F_j . If the 32 entries in the table sum to 256, what is the value of n?

	F_1	F_2	F_3	F_4
S_1				
S_2				
S_3				
S_4				
S_5				
S_6				
S_7				
S_8				

PART C: Question #1 (10 marks)

ABC is a triangle with coordinates $A=(2,6)$, $B=(0,0)$, and $C=(14,0)$.



(a) Let P be the midpoint of AB. Determine the equation of the line perpendicular to AB passing through P.
(b) Let Q be the point on line BC for which PQ is perpendicular to AB. Determine the length of AQ.
(c) There is a (unique) circle passing through the points A, B, and C. Determine the radius of this circle.

PART C: Question #2 (10 marks)

Charlotte writes a test consisting of 100 questions, where the answer to each question is either TRUE or FALSE. Charlotte's teacher announces that for every five *consecutive* questions on the test, the answers to *exactly* three of them are TRUE. Just before the test starts, the teacher whispers to Charlotte that the answers to the first and last questions are both FALSE.

(a) Determine the number of questions for which the correct answer is TRUE.

(b) What is the correct answer to the sixth question on the test?

(c) Explain how Charlotte can correctly answer all 100 questions on the test.

PART C: Question #3 (10 marks)

Let n be a positive integer. A row of $n+1$ squares is written from left to right, numbered 0, 1, 2, …, n, as shown.

0	1	2	$\cdots$	n

Two frogs, named Alphonse and Beryl, begin a race starting at square 0. For each second that passes, Alphonse and Beryl make a jump to the right according to the following rules: if there are at least eight squares to the right of Alphonse, then Alphonse jumps eight squares to the right. Otherwise, Alphonse jumps one square to the right. If there are at least seven squares to the right of Beryl, then Beryl jumps seven squares to the right. Otherwise, Beryl jumps one square to the right. Let $A(n)$ and $B(n)$ respectively denote the number of seconds for Alphonse and Beryl to reach square n. For example, $A(40) = 5$ and $B(40) = 10$.

(a) Determine an integer $n>200$ for which $B(n) < A(n)$.

(b) Determine the largest integer n for which $B(n) \leq A(n)$.

PART C: Question #4 (10 marks)

Let $f(x) = x^2 - ax + b$, where a and b are positive integers.

(a) Suppose $a = 2$ and $b = 2$. Determine the set of real roots of $f(x) - x$, and the set of real roots of $f(f(x)) - x$.

(b) Determine the number of pairs of positive integers (a, b) with $1 \le a, b \le 2011$ for which every root of $f(f(x)) - x$ is an integer.

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