The Canadian Mathematical Society La Société mathématique du Canada

The Canadian Mathematical Society

in collaboration with

The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

presents the

Sun Life Financial Canadian Open Mathematics Challenge

Sun Life Financial

Wednesday, November 24, 2010

Time: $2\frac{1}{2}$ hours

Calculators are NOT permitted.

Do not open this booklet until instructed to do so.

There are two parts to this paper.

PART A

This part of the paper consists of 8 questions, each worth 5 marks. You can earn full value for each question by entering the correct answer(s) in the space provided. If your answer is incorrect, any work that you do will be considered for part marks, **provided that it is done in the space allocated** to that question in your answer booklet.

PART B

This part of the paper consists of 4 questions, each worth 10 marks. Finished solutions must be written in the appropriate location in the answer booklet. Rough work should be done separately. If you require extra pages for your finished solutions, paper will be provided by your supervising teacher. Any extra papers should be placed inside your answer booklet. Be sure to write your name and school name on any inserted pages.

Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

NOTES:

At the completion of the contest, insert the information sheet inside the answer booklet.

The names of top scoring competitors will be published on the Web sites of the CMS and CEMC.

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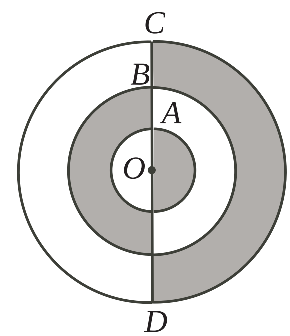
NOTE:
1. Please read the instructions on the front cover of this booklet.
2. Write solutions in the answer booklet provided.
3. It is expected that all calculations and answers will be expressed as exact numbers such as 4π, $2+\sqrt{7}$, etc., rather than as $12.566\ldots$ or $4.646\ldots$.
4. Calculators are **not** allowed.
5. Diagrams are not drawn to scale. They are intended as aids only.

PART A

1. Determine the integer equal to $\dfrac{(9+5)^2-(9-5)^2}{(9)(5)}$.

2. Determine all values of x for which $x-(8-x)=8-(x-8)$.

3. In the diagram, each of the three circles has centre O. Diameter CD of the largest circle passes through points B, A and O. The lengths of the radii of the circles are $OA=2$, $OB=4$, and $OC=6$. What is the area of the shaded region?

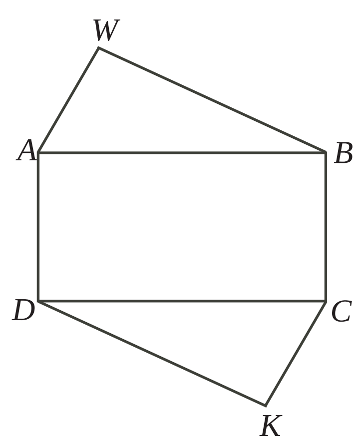


4. Determine the number of digits of the integer equal to $\dfrac{(3.1\times10^7)(8\times10^8)}{2\times10^3}$.

5. What point on the line with equation $y=x$ is closest to the point $P(-3,9)$?

6. On a calculus exam, the average of those who studied was 90% and the average of those who did not study was 40%. If the average of the entire class was 85%, what percentage of the class did not study?

7. In the diagram, rectangle $ABCD$ has $AB=20$ and $BC=10$. Points W and K are outside of the rectangle with $WA=KC=12$ and $WB=KD=16$. Determine the length of WK and express your answer in the form $WK=m\sqrt{n}$, where m and n are positive integers with $m>1$.



8. Determine all values of x for which $(x^2+3x+2)(x^2-2x-1)(x^2-7x+12)+24=0$.

PART B

1. In each part of this problem, each of the variables in the grid is to be replaced with an integer. The sum of the integers in a row is given to the right of the row. The sum of the integers in a column is given at the bottom of the column. For example, from the grid to the right we can conclude that $X+13=30$, $Y+11=23$, $X+Y=29$, and $13+11=24$.

X	13	30
Y	11	23
29	24	

   (a) Determine the value of C.

A	A	50
B	C	44
37	57	

   (b) Determine the value of n, the sum of the integers in the second column.

D	D	D	30
F	F	E	55
F	E	E	50
50	n	40	

   (c) Determine the value of $P+Q$.

P	Q	T	R	20
Q	P	T	R	20
R	R	R	T	33
T	T	T	R	19
20	20	19	33	

2. The parabola with equation $y = x^2 - 4x + 12$ intersects the line with equation $y = -2x + 20$ at points A and B.

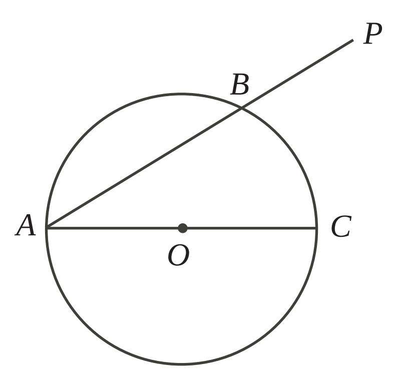
   (a) Determine the coordinates of the points A and B.

   (b) Determine the coordinates of the midpoint, M, of the segment AB.

   (c) A line parallel to the line with equation $y = -2x+20$ intersects the parabola at distinct points $P(p, p^2 - 4p + 12)$ and $Q(q, q^2 - 4q + 12)$. Prove that $p + q = 2$.

   (d) Point N is the midpoint of PQ. Explain why line segment MN is vertical.

3. In the diagram, the circle has centre O, diameter AC, and radius 1. A chord is drawn from A to an arbitrary point B (different from A) on the circle and extended to the point P with $BP = 1$. Thus P can take many positions. Let $\mathcal{S}$ be the set of points P.



   (a) Let U be a point in $\mathcal{S}$ for which UO is perpendicular to AC. Determine the length of UO.

   (b) Let V be a point in $\mathcal{S}$ for which VC is perpendicular to AC. Determine the length of VC.

   (c) Determine whether or not there is a circle on which all points of $\mathcal{S}$ lie.

4. Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x.
For example, $\lfloor 3.1 \rfloor = 3$ and $\lfloor -1.4 \rfloor = -2$.
For $x > 0$, define $f(x) = \left(x + \dfrac{1}{x}\right) - \left\lfloor x + \dfrac{1}{x} \right\rfloor$.

For example, $f\left(\frac{4}{9}\right) = \left(\frac{4}{9} + \frac{9}{4}\right) - \left\lfloor \frac{4}{9} + \frac{9}{4} \right\rfloor = \frac{97}{36} - 2 = \frac{25}{36}$.

(a) Determine all $x > 0$ so that $f(x) = x$.

(b) Suppose that $x = \dfrac{a}{a+1}$ for some positive integer $a > 1$.
Prove that $x \neq f(x)$, but that $f(x) = f(f(x))$.

(c) Prove that there are infinitely many rational numbers u so that

- $0 < u < 1$,
- u, $f(u)$ and $f(f(u))$ are all distinct, and
- $f(f(u)) = f(f(f(u)))$.