The Canadian Mathematical Society La Société mathématique du Canada

The Canadian Mathematical Society

in collaboration with

The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

presents the

Canadian Open Mathematics Challenge

Wednesday, November 22, 2006

Supported by: Sun Life Financial

Time: $2\frac{1}{2}$ hours

Calculators are NOT permitted.

Do not open this booklet until instructed to do so.
There are two parts to this paper.

PART A
This part of the paper consists of 8 questions, each worth 5 marks. You can earn full value for each question by entering the correct answer in the space provided. If you do not have the correct answer, any work you do in obtaining an answer will be considered for part marks, **provided that it is done in the space allocated** to that question in your answer booklet.

PART B
This part of the paper consists of 4 questions, each worth 10 marks. Finished solutions must be written in the appropriate location in the answer booklet. Rough work should be done separately. If you require extra pages for your finished solutions, paper will be provided by your supervising teacher. Any extra papers should be placed inside your answer booklet. Be sure to write your name and school name on any inserted pages.
Marks are awarded for completeness, clarity, and style of presentation. A correct solution poorly presented will not earn full marks.

NOTES:
At the completion of the contest, insert the information sheet inside the answer booklet.
The names of top scoring competitors will be published on the Web sites of the CMS and CEMC.

Canadian Open Mathematics Challenge

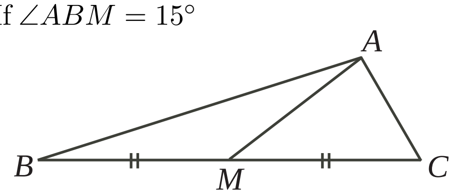
NOTE:	1.	Please read the instructions on the front cover of this booklet.
	2.	Write solutions in the answer booklet provided.
	3.	It is expected that all calculations and answers will be expressed as exact numbers such as 4π, $2+\sqrt{7}$, etc.
	4.	Calculators are **not** allowed.

PART A

1. What is the value of $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{5}\right)$?

2. If $f(2x+1)=(x-12)(x+13)$, what is the value of $f(31)$?

3. In $\triangle ABC$, M is the midpoint of BC, as shown. If $\angle ABM = 15^\circ$ and $\angle AMC = 30^\circ$, what is the size of $\angle BCA$?



4. Determine all solutions (x,y) to the system of equations

$$\begin{aligned}\frac{4}{x}+\frac{5}{y^2} &= 12\\ \frac{3}{x}+\frac{7}{y^2} &= 22\end{aligned}$$

5. In $\triangle ABC$, $BC=4$, $AB=x$, $AC=x+2$, and $\cos(\angle BAC)=\dfrac{x+8}{2x+4}$. Determine all possible values of x.

6. Determine the number of integers n that satisfy *all three* of the conditions below:
   - each digit of n is either 1 or 0,
   - n is divisible by 6, and
   - $0 < n < 10^7$.

7. Suppose n and D are integers with n positive and $0 \le D \le 9$. Determine n if $\dfrac{n}{810} = 0.\overline{9D5} = 0.9D59D59D5\ldots$.

8. What is the probability that 2 or more successive heads will occur at least once in 10 tosses of a fair coin?

PART B

1. Piotr places numbers on a 3 by 3 grid using the following rule, called "Piotr's Principle":

> For any three adjacent numbers in a horizontal, vertical or diagonal line, the middle number is always the average (mean) of its two neighbours.

(a) Using Piotr's principle, determine the missing numbers in the grid to the right. (You should fill in the missing numbers in the grid in your answer booklet.)

3		19
8		

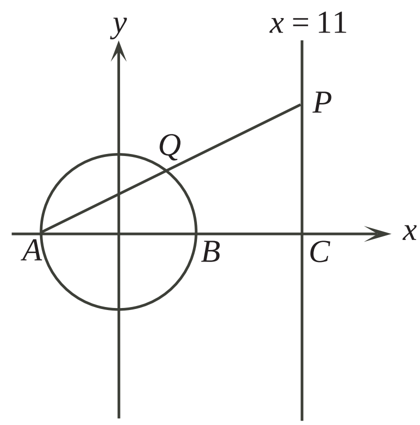
(b) Determine, with justification, the total of the nine numbers when the grid to the right is completed using Piotr's Principle.

x		
5		23

(c) Determine, with justification, the values of x and y when the grid to the right is completed using Piotr's Principle.

x	7	
9		y
		20

2. In the diagram, the circle $x^2 + y^2 = 25$ intersects the x-axis at points A and B. The line $x = 11$ intersects the x-axis at point C. Point P moves along the line $x = 11$ above the x-axis and AP intersects the circle at Q.

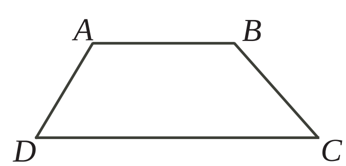


(a) Determine the coordinates of P when $\triangle AQB$ has maximum area. Justify your answer.

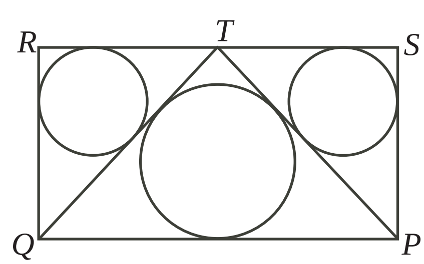
(b) Determine the coordinates of P when Q is the midpoint of AP. Justify your answer.

(c) Determine the coordinates of P when the area of $\triangle AQB$ is $\frac{1}{4}$ of the area of $\triangle APC$. Justify your answer.

3. (a) In the diagram, trapezoid $ABCD$ has parallel sides AB and DC of lengths 10 and 20, respectively. Also, the length of AD is 6 and the length of BC is 8. Determine the area of trapezoid $ABCD$.



(b) In the diagram, $PQRS$ is a rectangle and T is the midpoint of RS. The inscribed circles of $\triangle PTS$ and $\triangle RTQ$ each have radius 3. The inscribed circle of $\triangle QPT$ has radius 4. Determine the dimensions of rectangle $PQRS$.



4. (a) Determine, with justification, the fraction $\dfrac{p}{q}$, where p and q are positive integers and $q < 100$, that is closest to, but not equal to, $\frac{3}{7}$.

   (b) The *baseball sum* of two rational numbers $\dfrac{a}{b}$ and $\dfrac{c}{d}$ is defined to be $\dfrac{a+c}{b+d}$. (A *rational number* is a fraction whose numerator and denominator are both integers and whose denominator is not equal to 0.) Starting with the rational numbers $\frac{0}{1}$ and $\frac{1}{1}$ as Stage 0, the baseball sum of each consecutive pair of rational numbers in a stage is inserted between the pair to arrive at the next stage. The first few stages of this process are shown below:

STAGE 0:	$\frac{0}{1}$								$\frac{1}{1}$
STAGE 1:	$\frac{0}{1}$				$\frac{1}{2}$				$\frac{1}{1}$
STAGE 2:	$\frac{0}{1}$		$\frac{1}{3}$		$\frac{1}{2}$		$\frac{2}{3}$		$\frac{1}{1}$
STAGE 3:	$\frac{0}{1}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{1}{1}$

   Prove that

   (i) no rational number will be inserted more than once,

   (ii) no inserted fraction is reducible, and

   (iii) every rational number between 0 and 1 will be inserted in the pattern at some stage.