The Canadian Mathematical Society

in collaboration with

The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

The
Canadian Open Mathematics Challenge

Wednesday, November 28, 2001

Time: $2\frac{1}{2}$ hours

Calculators are NOT permitted.

Do not open this booklet until instructed to do so.
There are two parts to the paper.

PART A
This part of the paper consists of 8 questions, each worth 5 marks. You can earn full value for each question by entering the correct answer in the space provided. Any work you do in obtaining an answer will be considered for part marks if you do not have the correct answer, **provided that it is done in the space allocated** to that question in your answer booklet.

PART B
This part of the paper consists of 4 questions, each worth 10 marks. Finished solutions must be written in the appropriate location in the answer booklet. Rough work should be done separately. If you require extra pages for your finished solutions, paper will be provided by your supervising teacher. Any extra papers should be placed inside your answer booklet.
Marks are awarded for completeness, clarity, and style of presentation. A correct solution poorly presented will not earn full marks.

NOTE: At the completion of the contest, insert the information sheet inside the answer booklet.

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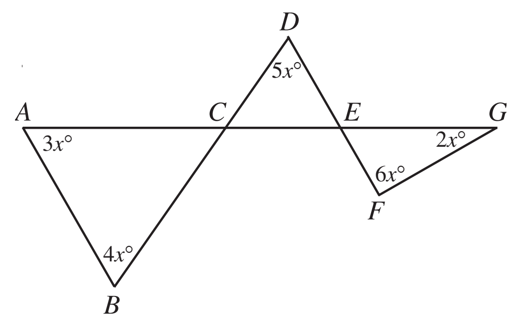
NOTE: 1. Please read the instructions on the front cover of this booklet.
2. Write solutions in the answer booklet provided.
3. It is expected that all calculations and answers will be expressed as exact numbers such as $4\pi, 2+\sqrt{7}$, etc.
4. Calculators are **not** allowed.

1. An operation "∇" is defined by $a \nabla b = a^2 + 3^b$.
   What is the value of $(2 \nabla 0) \nabla (0 \nabla 1)$?

2. In the given diagram, what is the value of x?

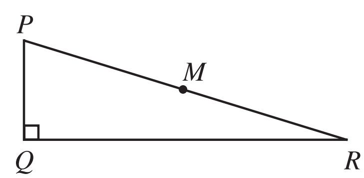


3. A regular hexagon is a six-sided figure which has all of its angles equal and all of its side lengths equal. If P and Q are points on a regular hexagon which has a side length of 1, what is the maximum possible length of the line segment PQ?

4. Solve for x:

$$2\left(2^{2x}\right) = 4^x + 64.$$

5. Triangle PQR is right-angled at Q and has side lengths $PQ = 14$ and $QR = 48$. If M is the midpoint of PR, determine the cosine of $\angle MQP$.



6. The sequence of numbers $t_1, t_2, t_3, \ldots$ is defined by $t_1 = 2$ and $t_{n+1} = \dfrac{t_n - 1}{t_n + 1}$, for every positive integer n.

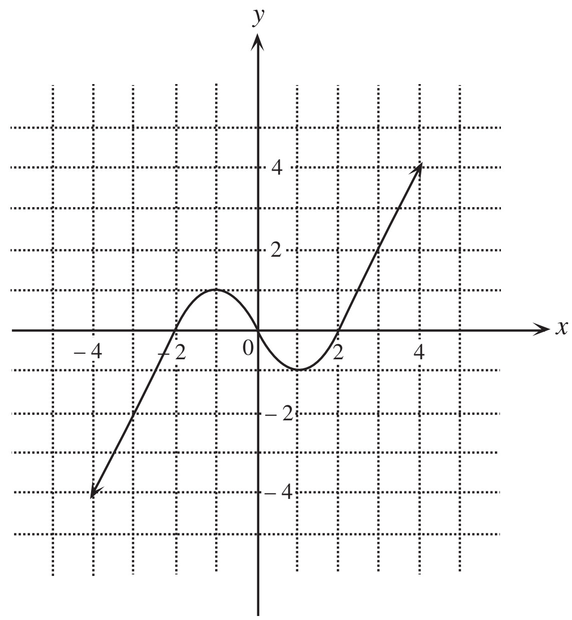
   Determine the numerical value of t_{999}.

7. If a can be any positive integer and

$$\begin{aligned} 2x + a &= y \\ a + y &= x \\ x + y &= z \end{aligned}$$

   determine the maximum possible value for $x + y + z$.

8. The graph of the function $y = g(x)$ is shown. Determine the number of solutions of the equation $\left| |g(x)| - 1 \right| = \frac{1}{2}$.



PART B

1. The triangular region T has its vertices determined by the intersections of the three lines $x + 2y = 12$, $x = 2$ and $y = 1$.
   (a) Determine the coordinates of the vertices of T, and show this region on the grid provided.
   (b) The line $x + y = 8$ divides the triangular region T into a quadrilateral Q and a triangle R. Determine the coordinates of the vertices of the quadrilateral Q.
   (c) Determine the area of the quadrilateral Q.

2. (a) Alphonse and Beryl are playing a game, starting with a pack of 7 cards. Alphonse begins by discarding at least one but not more than half of the cards in the pack. He then passes the remaining cards in the pack to Beryl. Beryl continues the game by discarding at least one but not more than half of the remaining cards in the pack. The game continues in this way with the pack being passed back and forth between the two players. The loser is the player who, at the beginning of his or her turn, receives only one card. Show, with justification, that there is always a winning strategy for Beryl.
   (b) Alphonse and Beryl now play a game with the same rules as in (a), except this time they start with a pack of 52 cards, and Alphonse goes first again. As in (a), a player on his or her turn must discard at least one and not more than half of the remaining cards from the pack. Is there a strategy that Alphonse can use to be guaranteed that he will win? (Provide justification for your answer.)

3. (a) If $f(x) = x^2 + 6x + c$, where c is an integer, prove that $f(0) + f(-1)$ is odd.
   (b) Let $g(x) = x^3 + px^2 + qx + r$, where p, q and r are integers. Prove that if $g(0)$ and $g(-1)$ are both odd, then the equation $g(x) = 0$ cannot have three integer roots.

4. Triangle ABC is isosceles with $AB = AC = 5$ and $BC = 6$. Point D lies on AC and P is the point on BD so that $\angle APC = 90°$. If $\angle ABP = \angle BCP$, determine the ratio $AD:DC$.