Canadian Mathematical Olympiad
1994

PROBLEM 1

Evaluate the sum

$$\sum_{n=1}^{1994}(-1)^n\frac{n^2+n+1}{n!}.$$

PROBLEM 2

Show that every positive integral power of $\sqrt{2}-1$ is of the form $\sqrt{m}-\sqrt{m-1}$ for some positive integer m. (e.g. $(\sqrt{2}-1)^2 = 3-2\sqrt{2} = \sqrt{9}-\sqrt{8}$).

PROBLEM 3

Twenty-five men sit around a circular table. Every hour there is a vote, and each must respond *yes* or *no*. Each man behaves as follows: on the n^{th} vote, if his response is the same as the response of at least one of the two people he sits between, then he will respond the same way on the $(n+1)^{th}$ vote as on the n^{th} vote; but if his response is different from that of both his neighbours on the n-th vote, then his response on the $(n+1)$-th vote will be different from his response on the n^{th} vote. Prove that, however everybody responded on the first vote, there will be a time after which nobody's response will ever change.

PROBLEM 4

Let AB be a diameter of a circle Ω and P be any point *not* on the line through A and B. Suppose the line through P and A cuts Ω again in U, and the line through P and B cuts Ω again in V. (Note that in case of tangency U may coincide with A or V may coincide with B. Also, if P is on Ω then $P=U=V$.) Suppose that $|PU| = s|PA|$ and $|PV| = t|PB|$ for some nonnegative real numbers s and t. Determine the cosine of the angle APB in terms of s and t.

PROBLEM 5

Let ABC be an acute angled triangle. Let AD be the altitude on BC, and let H be any interior point on AD. Lines BH and CH, when extended, intersect AC and AB at E and F, respectively. Prove that $\angle EDH = \angle FDH$.