Canadian Mathematical Olympiad
1989

Problem 1

The integers $1, 2, \ldots, n$ are placed in order so that each value is either strictly bigger than all the preceding values or is strictly smaller than all preceding values. In how many ways can this be done?

Problem 2

Let ABC be a right angled triangle of area 1. Let $A'B'C'$ be the points obtained by reflecting A, B, C respectively, in their opposite sides. Find the area of $\triangle A'B'C'$.

Problem 3

Define $\{a_n\}_{n=1}$ as follows: $a_1 = 1989^{1989}$; a_n, $n > 1$, is the sum of the digits of a_{n-1}. What is the value of a_5?

Problem 4

There are 5 monkeys and 5 ladders and at the top of each ladder there is a banana. A number of ropes connect the ladders, each rope connects two ladders. No two ropes are attached to the same rung of the same ladder. Each monkey starts at the bottom of a different ladder. The monkeys climb up the ladders but each time they encounter a rope they climb along it to the ladder at the other end of the rope and then continue climbing upwards. Show that, no matter how many ropes there are, each monkey gets a banana.

Problem 5

Given the numbers $1, 2, 2^2, \ldots, 2^{n-1}$. For a specific permutation $\sigma = X_1, X_2, \ldots, X_n$ of these numbers we define $S_1(\sigma) = X_1$, $S_2(\sigma) = X_1 + X_2$, $S_3(\sigma) = X_1 + X_2 + X_3, \ldots$ and $Q(\sigma) = S_1(\sigma)S_2(\sigma)\cdots S_n(\sigma)$. Evaluate $\sum 1/Q(\sigma)$ where the sum is taken over all possible permutations.