Canadian Mathematical Olympiad
1988

Problem 1

For what values of b do the equations: $1988x^2 + bx + 8891 = 0$ and $8891x^2 + bx + 1988 = 0$ have a common root?

Problem 2

A house is in the shape of a triangle, perimeter P metres and area A square metres. The garden consists of all the land within 5 metres of the house. How much land do the garden and house together occupy?

Problem 3

Suppose that S is a finite set of at least five points in the plane; some are coloured red, the others are coloured blue. No subset of three or more similarly coloured points is collinear. Show that there is a triangle

(i) whose vertices are all the same colour, and

(ii) at least one side of the triangle does not contain a point of the opposite colour.

Problem 4

Let $x_{n+1} = 4x_n - x_{n-1}$, $x_0 = 0$, $x_1 = 1$, and $y_{n+1} = 4y_n - y_{n-1}$, $y_0 = 1$, $y_1 = 2$. Show for all $n \geq 0$ that $y_n^2 = 3x_n^2 + 1$.

Problem 5

Let $S = \{a_1, a_2, \ldots, a_r\}$ denote a sequence of integers. For each non-empty subsequence A of S, we define $p(A)$ to be the product of all the integers in A. Let $m(S)$ be the arithmetic average of $p(A)$ over all non-empty subsets A of S. If $m(S) = 13$ and if $m(S \cup \{a_{r+1}\}) = 49$ for some positive integer a_{r+1}, determine the values of $a_1, a_2, \ldots, a_r$ and a_{r+1}.