Canadian Mathematical Olympiad
1981

PROBLEM 1

For any real number t, denote by $[t]$ the greatest integer which is less than or equal to t. For example: $[8] = 8$, $[\pi] = 3$ and $[-5/2] = -3$. Show that the equation

$$[x] + [2x] + [4x] + [8x] + [16x] + [32x] = 12345$$

has no real solution.

PROBLEM 2

Given a circle of radius r and a tangent line ℓ to the circle through a given point P on the circle. From a variable point R on the circle, a perpendicular RQ is drawn to ℓ with Q on ℓ. Determine the maximum of the area of triangle PQR.

PROBLEM 3

Given a finite collection of lines in a plane P, show that it is possible to draw an arbitrarily large circle in P which does not meet any of them. On the other hand, show that it is possible to arrange an infinite sequence of lines (first line, second line, third line, *etc.*) in P so that every circle in P meets at least one of the lines. (A point is not considered to be a circle.)

PROBLEM 4

$P(x)$ and $Q(x)$ are two polynomials that satisfy the identity $P\big(Q(x)\big) \equiv Q\big(P(x)\big)$ for real numbers x. If the equation $P(x) = Q(x)$ has no real solution, show that the equation $P\big(P(x)\big) = Q\big(Q(x)\big)$ also has no real solution.

PROBLEM 5

Eleven theatrical groups participated in a festival. Each day, some of the groups were scheduled to perform while the remaining groups joined the general audience. At the conclusion of the festival, each group had seen, during its days off, at least one performance of every other group. At least how many days did the festival last?