Canadian Mathematical Olympiad
1976

PROBLEM 1

Given four weights in geometric progression and an equal arm balance, show how to find the heaviest weight using the balance only twice.

PROBLEM 2

Suppose

$$n(n+1)a_{n+1} = n(n-1)a_n - (n-2)a_{n-1}$$

for every positive integer $n \geq 1$.
Given that $a_0 = 1$, $a_1 = 2$, find

$$\frac{a_0}{a_1} + \frac{a_1}{a_2} + \frac{a_2}{a_3} + \cdots + \frac{a_{50}}{a_{51}}.$$

PROBLEM 3

Two grade seven students were allowed to enter a chess tournament otherwise composed of grade eight students. Each contestant played once with each other contestant and received one point for a win, one half point for a tie and zero for a loss. The two grade seven students together gained a total of eight points and each grade eight student scored the same number of points as his classmates. How many students from grade eight participated in the chess tournament? Is the solution unique?

PROBLEM 4

Let AB be a diameter of a circle, C be any fixed point between A and B on this diameter, and Q be a variable point on the circumference of the circle. Let P be the point on the line determined by Q and C for which $\frac{AC}{CB} = \frac{QC}{CP}$. Describe, with proof, the locus of the point P.

PROBLEM 5

Prove that a positive integer is a sum of at least two consecutive positive integers if and only if it is not a power of two.

PROBLEM 6

If A, B, C, D are four points in space, such that

$$\angle ABC = \angle BCD = \angle CDA = \angle DAB = \pi/2,$$

prove that A, B, C, D lie in a plane.

Problem 7

Let $P(x,y)$ be a polynomial in two variables x, y such that $P(x,y) = P(y,x)$ for every x, y (for example, the polynomial $x^2-2xy+y^2$ satisfies this condition). Given that $(x-y)$ is a factor of $P(x,y)$, show that $(x-y)^2$ is a factor of $P(x,y)$.

Problem 8

Each of the 36 line segments joining 9 distinct points on a circle is coloured either red or blue. Suppose that each triangle determined by 3 of the 9 points contains at least one red side. Prove that there are four points such that the 6 segments connecting them are all red.