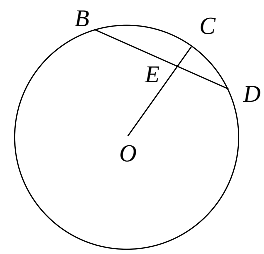
Canadian Mathematical Olympiad
1971

PROBLEM 1



DEB is a chord of a circle such that $DE = 3$ and $EB = 5$. Let O be the centre of the circle. Join OE and extend OE to cut the circle at C. (See diagram). Given $EC = 1$, find the radius of the circle.

PROBLEM 2

Let x and y be positive real numbers such that $x + y = 1$. Show that

$$\left(1+\frac{1}{x}\right)\left(1+\frac{1}{y}\right) \geq 9.$$

PROBLEM 3

$ABCD$ is a quadrilateral with $AD = BC$. If $\angle ADC$ is greater than $\angle BCD$, prove that $AC > BD$.

PROBLEM 4

Determine all real numbers a such that the two polynomials x^2+ax+1 and x^2+x+a have at least one root in common.

PROBLEM 5

Let

$$p(x) = a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n,$$

where the coefficients a_i are integers. If $p(0)$ and $p(1)$ are both odd, show that $p(x)$ has no integral roots.

PROBLEM 6

Show that, for all integers n, $n^2 + 2n + 12$ is not a multiple of 121.

PROBLEM 7

Let n be a five digit number (whose first digit is non-zero) and let m be the four digit number formed from n by deleting its middle digit. Determine all n such that n/m is an integer.

PROBLEM 8

A regular pentagon is inscribed in a circle of radius r. P is any point inside the pentagon. Perpendiculars are dropped from P to the sides, or the sides produced, of the pentagon.

a) Prove that the sum of the lengths of these perpendiculars is constant.
b) Express this constant in terms of the radius r.

Problem 9

Two flag poles of heights h and k are situated $2a$ units apart on a level surface. Find the set of all points on the surface which are so situated that the angles of elevation of the tops of the poles are equal.

Problem 10

Suppose that n people each know exactly one piece of information, and all n pieces are different. Every time person A phones person B, A tells B everything that A knows, while B tells A nothing. What is the minimum number of phone calls between pairs of people needed for everyone to know everything? Prove your answer is a minimum.