The 2020 Canadian Junior Mathematical Olympiad

A competition of the Canadian Mathematical Society and supported by the Actuarial Profession.

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Official Problem Set

1. Let $a_1, a_2, a_3, \ldots$ be a sequence of positive real numbers that satisfies

$$a_1 = 1 \quad \text{and} \quad a_{n+1}^2 + a_{n+1} \;=\; a_n \quad \text{for every natural number } n.$$

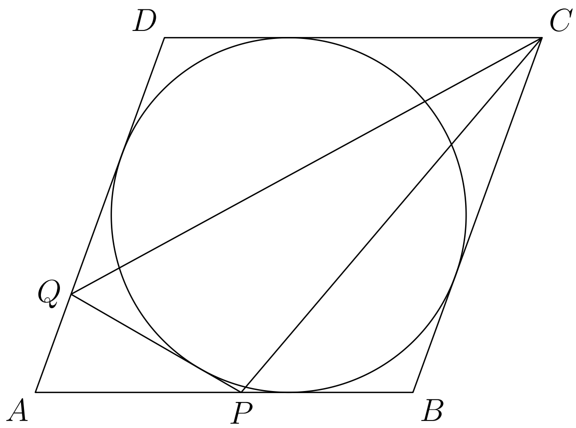
Prove that $a_n \geq \frac{1}{n}$ for every natural number n.

2. Ziquan makes a drawing in the plane for art class. He starts by placing his pen at the origin, and draws a series of line segments, such that the n^{th} line segment has length n. He is not allowed to lift his pen, so that the end of the n^{th} segment is the start of the $(n+1)^{th}$ segment. Line segments drawn are allowed to intersect and even overlap previously drawn segments.

After drawing a finite number of line segments, Ziquan stops and hands in his drawing to his art teacher. He passes the course if the drawing he hands in is an N by N square, for some positive integer N, and he fails the course otherwise. Is it possible for Ziquan to pass the course?

3. Let S be a set of $n \geq 3$ positive real numbers. Show that the largest possible number of distinct integer powers of three that can be written as the sum of three distinct elements of S is $n - 2$.

4. A circle is inscribed in a rhombus $ABCD$. Points P and Q vary on line segments $\overline{AB}$ and $\overline{AD}$, respectively, so that $\overline{PQ}$ is tangent to the circle. Show that for all such line segments $\overline{PQ}$, the area of triangle CPQ is constant.



5. A purse contains a finite number of coins, each with distinct positive integer values. Is it possible that there are exactly 2020 ways to use coins from the purse to make the value 2020?

Important!

Please do not discuss this problem set online for at least 24 hours.

