International Mathematical Talent Search – Round 9

Problem 1/9. An $m \times n$ grid is placed so that it has it's corners at $(0, 0)$ and (m, n). A legal move is defined as a move either one unit in the positive y direction or one unit in the positive x direction. The point (i, j), where $0 \leq i \leq m$ and $0 \leq j \leq n$, is removed from the grid so that it is no longer possible to pass through this point on the way to (m, n). How many possible paths are there from $(0, 0)$ to (m, n)?

Problem 2/9. Given a point P and two straight line segments on a rectangular piece of paper in such a way that the intersection point Q of the straight lines does not lie on the paper. How can we construct the straight line PQ with the help of a ruler if we are allowed to draw only within the limits of the paper?

Problem 3/9. A convex polygon has 1993 vertices which are colored so that neighboring vertices are of different colors. Prove that one can divide the polygon into triangles with non-intersecting diagonals whose endpoints are of different colors.

Problem 4/9. A triangle is called Heronian if its sides and area are integers. Determine all five Heronian triangles whose perimeter is numerically the same as its area.

Problem 5/9. A set of five "Trick Math Cubes" is shown schematically on the right. A "magician" asks you to roll them and to add the five numbers on top of them. He starts adding them at the same time, and writes down the correct answer on a piece of paper long before you are finished with the task. How does he do it? Expose and explain this trick.